

General Certificate of Education
June 2005
Advanced Subsidiary Examination



MATHEMATICS
Unit Pure Core 1

MPC1

Tuesday 7 June 2005 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
 - the **blue** AQA booklet of formulae and statistical tables.
- You must **not** use a calculator.



Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information

- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

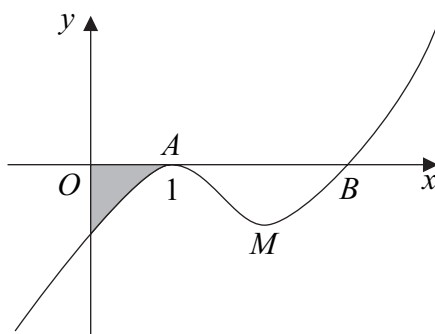
Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1** The point A has coordinates $(6, 5)$ and the point B has coordinates $(2, -1)$.
- (a) Find the coordinates of the midpoint of AB . *(2 marks)*
- (b) Show that AB has length $k\sqrt{13}$, where k is an integer. *(3 marks)*
- (c) (i) Find the gradient of the line AB . *(2 marks)*
- (ii) Hence, or otherwise, show that the line AB has equation $3x - 2y = 8$. *(2 marks)*
- (d) The line AB intersects the line with equation $2x + y = 10$ at the point C . Find the coordinates of C . *(3 marks)*
- 2** (a) Express $x^2 - 6x + 16$ in the form $(x - p)^2 + q$. *(2 marks)*
- (b) A curve has equation $y = x^2 - 6x + 16$.
- Using your answer from part (a), or otherwise:
- (i) find the coordinates of the vertex (minimum point) of the curve; *(2 marks)*
- (ii) sketch the curve, indicating the value where the curve crosses the y -axis; *(2 marks)*
- (iii) state the equation of the line of symmetry of the curve. *(1 mark)*
- (c) Describe geometrically the transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 - 6x + 16$. *(3 marks)*
- 3** A circle has centre $C(2, -1)$ and radius 5. The point P has coordinates $(6, 2)$.
- (a) Write down an equation of the circle. *(3 marks)*
- (b) Verify that the point P lies on the circle. *(2 marks)*
- (c) Find the gradient of the line CP . *(2 marks)*
- (d) (i) Find the gradient of a line which is perpendicular to CP . *(2 marks)*
- (ii) Hence find an equation for the tangent to the circle at the point P . *(1 mark)*

- 4 The curve with equation $y = x^3 - 5x^2 + 7x - 3$ is sketched below.



The curve touches the x -axis at the point $A(1, 0)$ and cuts the x -axis at the point B .

- (a) (i) Use the factor theorem to show that $x - 3$ is a factor of

$$p(x) = x^3 - 5x^2 + 7x - 3 \quad (2 \text{ marks})$$

- (ii) Hence find the coordinates of B . (1 mark)

- (b) The point M , shown on the diagram, is a minimum point of the curve with equation $y = x^3 - 5x^2 + 7x - 3$.

- (i) Find $\frac{dy}{dx}$. (2 marks)

- (ii) Hence determine the x -coordinate of M . (3 marks)

- (c) Find the value of $\frac{d^2y}{dx^2}$ when $x = 1$. (2 marks)

- (d) (i) Find $\int (x^3 - 5x^2 + 7x - 3) dx$. (4 marks)

- (ii) Hence determine the area of the shaded region bounded by the curve and the coordinate axes. (4 marks)

- 5 Express each of the following in the form $m + n\sqrt{3}$, where m and n are integers:

- (a) $(\sqrt{3} + 1)^2$; (2 marks)

- (b) $\frac{\sqrt{3} + 1}{\sqrt{3} - 1}$. (3 marks)

- 6 The cubic polynomial $p(x)$ is given by $p(x) = (x - 2)(x^2 + x + 3)$.
- (a) Show that $p(x)$ can be written in the form $x^3 + ax^2 + bx - 6$, where a and b are constants whose values are to be found. *(2 marks)*
- (b) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x + 1$. *(2 marks)*
- (c) Prove that the equation $(x - 2)(x^2 + x + 3) = 0$ has only one real root and state its value. *(3 marks)*

7 Solve each of the following inequalities:

- (a) $3(x - 1) > 3 - 5(x + 6)$; *(3 marks)*
- (b) $x^2 - x - 6 < 0$. *(4 marks)*

8 A line has equation $y = mx - 1$, where m is a constant.

A curve has equation $y = x^2 - 5x + 3$.

- (a) Show that the x -coordinate of any point of intersection of the line and the curve satisfies the equation

$$x^2 - (5 + m)x + 4 = 0 \quad (1 \text{ mark})$$

- (b) Find the values of m for which the equation $x^2 - (5 + m)x + 4 = 0$ has equal roots. *(4 marks)*

- (c) Describe geometrically the situation when m takes either of the values found in part (b). *(1 mark)*

END OF QUESTIONS