

General Certificate of Education
January 2007
Advanced Subsidiary Examination



MATHEMATICS
Unit Pure Core 1

MPC1

Wednesday 10 January 2007 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
 - the **blue** AQA booklet of formulae and statistical tables.
- You must **not** use a calculator.



Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The polynomial $p(x)$ is given by

$$p(x) = x^3 - 4x^2 - 7x + k$$

where k is a constant.

- (a) (i) Given that $x + 2$ is a factor of $p(x)$, show that $k = 10$. (2 marks)
- (ii) Express $p(x)$ as the product of three linear factors. (3 marks)
- (b) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x - 3$. (2 marks)
- (c) Sketch the curve with equation $y = x^3 - 4x^2 - 7x + 10$, indicating the values where the curve crosses the x -axis and the y -axis. (You are **not** required to find the coordinates of the stationary points.) (4 marks)

2 The line AB has equation $3x + 5y = 8$ and the point A has coordinates $(6, -2)$.

- (a) (i) Find the gradient of AB . (2 marks)
- (ii) Hence find an equation of the straight line which is perpendicular to AB and which passes through A . (3 marks)
- (b) The line AB intersects the line with equation $2x + 3y = 3$ at the point B . Find the coordinates of B . (3 marks)
- (c) The point C has coordinates $(2, k)$ and the distance from A to C is 5. Find the **two** possible values of the constant k . (3 marks)

3 (a) Express $\frac{\sqrt{5} + 3}{\sqrt{5} - 2}$ in the form $p\sqrt{5} + q$, where p and q are integers. (4 marks)

- (b) (i) Express $\sqrt{45}$ in the form $n\sqrt{5}$, where n is an integer. (1 mark)
- (ii) Solve the equation

$$x\sqrt{20} = 7\sqrt{5} - \sqrt{45}$$

giving your answer in its simplest form. (3 marks)

4 A circle with centre C has equation $x^2 + y^2 + 2x - 12y + 12 = 0$.

(a) By completing the square, express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2 \quad (3 \text{ marks})$$

(b) Write down:

(i) the coordinates of C ; (1 mark)

(ii) the radius of the circle. (1 mark)

(c) Show that the circle does **not** intersect the x -axis. (2 marks)

(d) The line with equation $x + y = 4$ intersects the circle at the points P and Q .

(i) Show that the x -coordinates of P and Q satisfy the equation

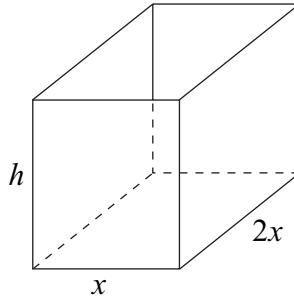
$$x^2 + 3x - 10 = 0 \quad (3 \text{ marks})$$

(ii) Given that P has coordinates $(2, 2)$, find the coordinates of Q . (2 marks)

(iii) Hence find the coordinates of the midpoint of PQ . (2 marks)

Turn over for the next question

- 5 The diagram shows an **open-topped** water tank with a horizontal rectangular base and four vertical faces. The base has width x metres and length $2x$ metres, and the height of the tank is h metres.



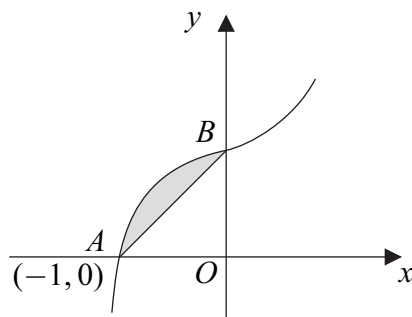
The combined internal surface area of the base and four vertical faces is 54 m^2 .

- (a) (i) Show that $x^2 + 3xh = 27$. (2 marks)
- (ii) Hence express h in terms of x . (1 mark)
- (iii) Hence show that the volume of water, $V \text{ m}^3$, that the tank can hold when full is given by

$$V = 18x - \frac{2x^3}{3} \quad (1 \text{ mark})$$

- (b) (i) Find $\frac{dV}{dx}$. (2 marks)
- (ii) Verify that V has a stationary value when $x = 3$. (2 marks)
- (c) Find $\frac{d^2V}{dx^2}$ and hence determine whether V has a maximum value or a minimum value when $x = 3$. (2 marks)

6 The curve with equation $y = 3x^5 + 2x + 5$ is sketched below.



The curve cuts the x -axis at the point $A(-1, 0)$ and cuts the y -axis at the point B .

- (a) (i) State the coordinates of the point B and hence find the area of the triangle AOB , where O is the origin. *(3 marks)*
- (ii) Find $\int (3x^5 + 2x + 5) dx$. *(3 marks)*
- (iii) Hence find the area of the shaded region bounded by the curve and the line AB . *(4 marks)*
- (b) (i) Find the gradient of the curve with equation $y = 3x^5 + 2x + 5$ at the point $A(-1, 0)$. *(3 marks)*
- (ii) Hence find an equation of the tangent to the curve at the point A . *(1 mark)*

7 The quadratic equation $(k + 1)x^2 + 12x + (k - 4) = 0$ has real roots.

- (a) Show that $k^2 - 3k - 40 \leq 0$. *(3 marks)*
- (b) Hence find the possible values of k . *(4 marks)*

END OF QUESTIONS

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