General Certificate of Education
June 2006
Advanced Level Examination

**MATHEMATICS** **MM2B**
**Unit Mechanics 2B**

Tuesday 6 June 2006 1.30 pm to 3.00 pm

**For this paper you must have:**
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

**Information**
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Mechanics 2B has a **written paper only**.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

**1** A particle moves in a horizontal plane, in which the unit vectors **i** and **j** are directed east and north respectively. At time $t$ seconds, its position vector, **r** metres, is given by

$$\mathbf{r} = (2t^3 - t^2 + 6)\mathbf{i} + (8 - 4t^3 + t)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time $t$. *(3 marks)*

(b) (i) Find the velocity of the particle when $t = \frac{1}{3}$. *(2 marks)*

(ii) State the direction in which the particle is travelling at this time. *(1 mark)*

(c) Find the acceleration of the particle when $t = 4$. *(3 marks)*

(d) The mass of the particle is 6 kg. Find the magnitude of the resultant force on the particle when $t = 4$. *(3 marks)*

**2** A ball of mass 0.6 kg is thrown vertically upwards from ground level with an initial speed of $14\,\mathrm{m\,s^{-1}}$.

(a) Calculate the initial kinetic energy of the ball. *(2 marks)*

(b) Assuming that no resistance forces act on the ball, use an energy method to find the maximum height reached by the ball. *(3 marks)*

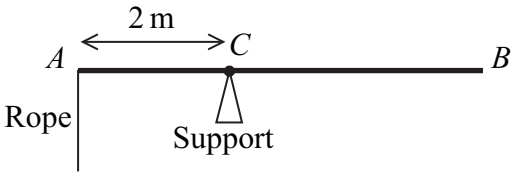
(c) An experiment is conducted to confirm the maximum height for the ball calculated in part (b). In this experiment the ball rises to a height of only 8 metres.

(i) Find the work done against the air resistance force that acts on the ball as it moves. *(3 marks)*

(ii) Assuming that the air resistance force is constant, find its magnitude. *(2 marks)*

(d) Explain why it is **not** realistic to model the air resistance as a constant force. *(1 mark)*

**3** The diagram shows a uniform rod, $AB$, of mass 10 kg and length 5 metres. The rod is held in equilibrium in a horizontal position, by a support at $C$ and a light vertical rope attached to $A$, where $AC$ is 2 metres.




(a) Draw and label a diagram to show the forces acting on the rod. *(1 mark)*

(b) Show that the tension in the rope is 24.5 N. *(3 marks)*

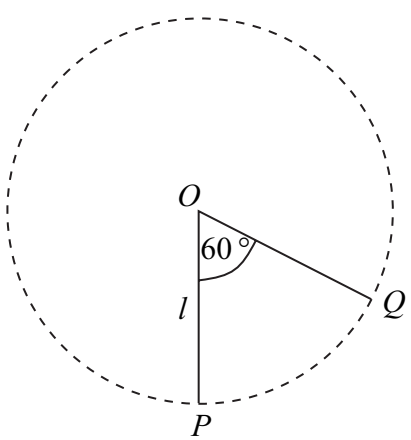
(c) A package of mass $m$ kg is suspended from $B$. The tension in the rope has to be doubled to maintain equilibrium.

(i) Find $m$. *(4 marks)*

(ii) Find the magnitude of the force exerted on the rod by the support. *(3 marks)*

(d) Explain how you have used the fact that the rod is uniform in your solution. *(1 mark)*

**4** A particle of mass $m$ is suspended from a fixed point $O$ by a light inextensible string of length $l$. The particle hangs in equilibrium at the point $P$ vertically below $O$. The particle is then set into motion with a horizontal velocity $U$ so that it moves in a complete vertical circle with centre $O$. The point $Q$ on the circle is such that $\angle POQ = 60°$, as shown in the diagram.

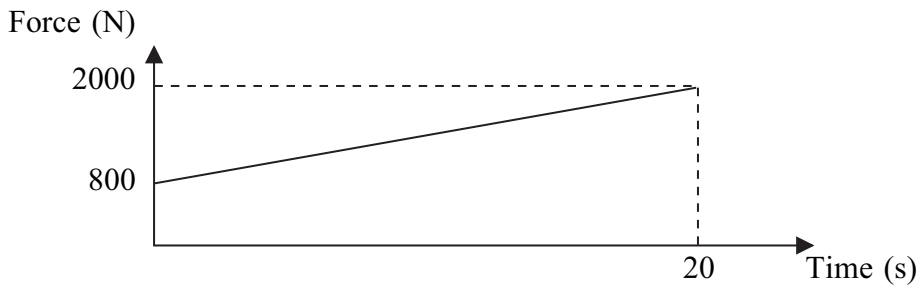



(a) Find, in terms of $g$, $l$ and $U$, the speed of the particle at $Q$. *(4 marks)*

(b) Find, in terms of $g$, $l$, $m$ and $U$, the tension in the string when the particle is at $Q$. *(5 marks)*

(c) Find, in terms of $g$, $l$, $m$ and $U$, the tension in the string when the particle returns to $P$. *(2 marks)*

**5** The graph shows a model for the resultant horizontal force on a car, which varies as it accelerates from rest for 20 seconds. The mass of the car is 1200 kg.




(a) The acceleration of the car at time $t$ seconds is $a\ \mathrm{m\,s^{-2}}$. Show that

$$a = \frac{2}{3} + \frac{t}{20}, \quad \text{for} \quad 0 \leqslant t \leqslant 20$$

*(5 marks)*

(b) Find an expression for the velocity of the car at time $t$. *(3 marks)*

(c) Find the distance travelled by the car in the 20 seconds. *(4 marks)*

(d) An alternative model assumes that the resultant force increases uniformly from 900 to 2100 newtons during the 20 seconds. Which term in your expression for the velocity would change as a result of this modification? Explain why. *(2 marks)*

**6** A car of mass 1200 kg travels round a roundabout on a horizontal, circular path at a constant speed of $14\ \mathrm{m\,s^{-1}}$. The radius of the circle is 50 metres. Assume that there is no resistance to the motion of the car and that the car can be modelled as a particle.

(a) A friction force, directed towards the centre of the roundabout, acts on the car as it moves. Show that the magnitude of this friction force is 4704 N. *(4 marks)*

(b) The coefficient of friction between the car and the road is $\mu$. Show that $\mu \geqslant 0.4$. *(3 marks)*

**7** A particle of mass 20 kg moves along a straight horizontal line. At time $t$ seconds the velocity of the particle is $v\ \mathrm{m\,s^{-1}}$. A resistance force of magnitude $10\sqrt{v}$ newtons acts on the particle while it is moving. At time $t = 0$ the velocity of the particle is $25\ \mathrm{m\,s^{-1}}$.

(a) Show that, at time $t$

$$v = \left(\frac{20 - t}{4}\right)^2$$

*(7 marks)*

(b) State the value of $t$ when the particle comes to rest. *(1 mark)*

**END OF QUESTIONS**