

General Certificate of Education
January 2007
Advanced Level Examination



MATHEMATICS
Unit Mechanics 2B

MM2B

Tuesday 16 January 2007 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information

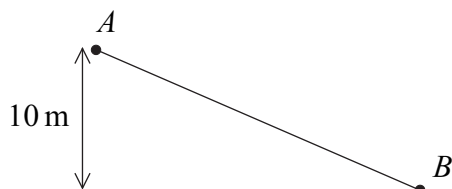
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Mechanics 2B has a **written paper only**.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 A child, of mass 35 kg, slides down a slide in a water park. The child, starting from rest, slides from the point A to the point B , which is 10 metres vertically below the level of A , as shown in the diagram.



- (a) In a simple model, all resistance forces are ignored.

Use an energy method to find the speed of the child at B . (3 marks)

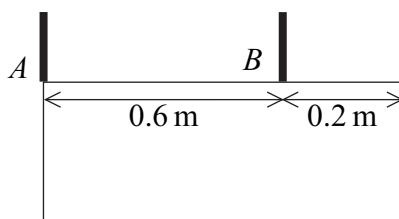
- (b) State one resistance force that has been ignored in answering part (a). (1 mark)

- (c) In fact, when the child slides down the slide, she reaches B with a speed of 12 m s^{-1} .

Given that the slide is 20 metres long and the sum of the resistance forces has a constant magnitude of F newtons, use an energy method to find the value of F .

(4 marks)

- 2 A hotel sign consists of a uniform rectangular lamina of weight W . The sign is suspended in equilibrium in a vertical plane by two vertical light chains attached to the sign at the points A and B , as shown in the diagram. The edge containing A and B is horizontal.



The tensions in the chains attached at A and B are T_A and T_B respectively.

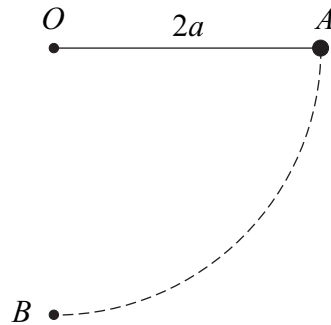
- (a) Draw a diagram to show the forces acting on the sign. (1 mark)

- (b) Find T_A and T_B in terms of W . (4 marks)

- (c) Explain how you have used the fact that the lamina is uniform in answering part (b).

(1 mark)

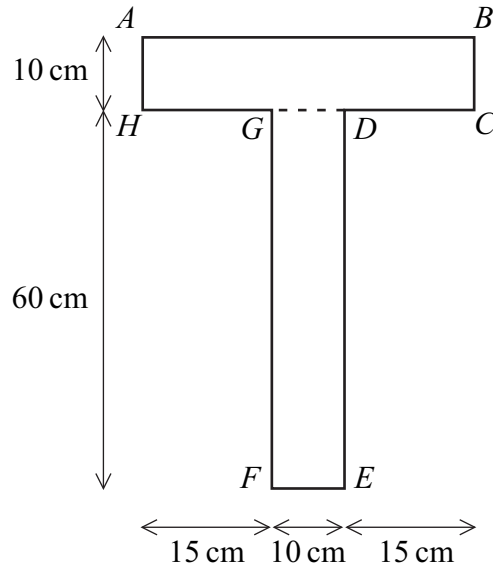
- 3 A light inextensible string has length $2a$. One end of the string is attached to a fixed point O and a particle of mass m is attached to the other end. Initially, the particle is held at the point A with the string taut and horizontal. The particle is then released from rest and moves in a circular path. Subsequently, it passes through the point B , which is directly below O . The points O , A and B are as shown in the diagram.



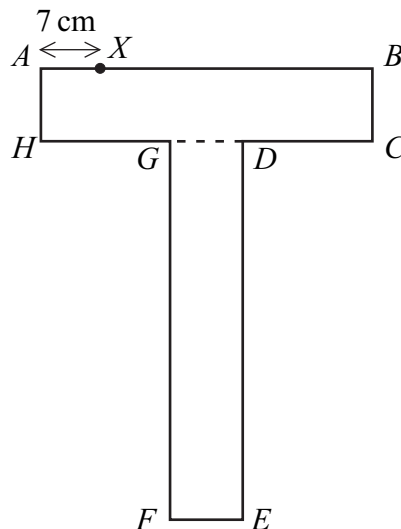
- (a) Show that the speed of the particle at B is $2\sqrt{ag}$. (3 marks)
- (b) Find the tension in the string as the particle passes through B . Give your answer in terms of m and g . (3 marks)

Turn over for the next question

- 4 A uniform T-shaped lamina is formed by rigidly joining two rectangles $ABCH$ and $DEFG$, as shown in the diagram.



- (a) Show that the centre of mass of the lamina is 26 cm from the edge AB . (4 marks)
- (b) Explain why the centre of mass of the lamina is 5 cm from the edge GF . (1 mark)
- (c) The point X is on the edge AB and is 7 cm from A , as shown in the diagram below.



The lamina is freely suspended from X and hangs in equilibrium.

Find the angle between the edge AB and the vertical, giving your answer to the nearest degree. (4 marks)

5 Tom is on a fairground ride.

Tom's position vector, \mathbf{r} metres, at time t seconds is given by

$$\mathbf{r} = 2 \cos t \mathbf{i} + 2 \sin t \mathbf{j} + (10 - 0.4t)\mathbf{k}$$

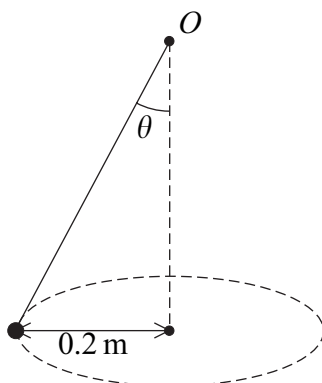
The perpendicular unit vectors \mathbf{i} and \mathbf{j} are in the horizontal plane and the unit vector \mathbf{k} is directed vertically upwards.

- (a) (i) Find Tom's position vector when $t = 0$. *(1 mark)*
- (ii) Find Tom's position vector when $t = 2\pi$. *(1 mark)*
- (iii) Write down the first **two** values of t for which Tom is directly below his starting point. *(2 marks)*
- (b) Find an expression for Tom's velocity at time t . *(3 marks)*
- (c) Tom has mass 25 kg.

Show that the resultant force acting on Tom during the motion has constant magnitude.
State the magnitude of the resultant force. *(5 marks)*

Turn over for the next question

- 6 A particle is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O . The particle is set into motion, so that it describes a horizontal circle whose centre is vertically below O . The angle between the string and the vertical is θ , as shown in the diagram.



- (a) The particle completes 40 revolutions every minute.

Show that the angular speed of the particle is $\frac{4\pi}{3}$ radians per second. *(2 marks)*

- (b) The radius of the circle is 0.2 metres.

Find, in terms of π , the magnitude of the acceleration of the particle. *(2 marks)*

- (c) The mass of the particle is m kg and the tension in the string is T newtons.

(i) Draw a diagram showing the forces acting on the particle. *(1 mark)*

(ii) Explain why $T \cos \theta = mg$. *(1 mark)*

(iii) Find the value of θ , giving your answer to the nearest degree. *(5 marks)*

7 A motorcycle has a maximum power of 72 kilowatts. The motorcycle and its rider are travelling along a straight horizontal road. When they are moving at a speed of $V \text{ m s}^{-1}$, they experience a total resistance force of magnitude kV newtons, where k is a constant.

(a) The maximum speed of the motorcycle and its rider is 60 m s^{-1} .

Show that $k = 20$.

(3 marks)

(b) When the motorcycle is travelling at 20 m s^{-1} , the rider allows the motorcycle to freewheel so that the only horizontal force acting is the resistance force. When the motorcycle has been freewheeling for t seconds, its speed is $v \text{ m s}^{-1}$ and the magnitude of the resistance force is $20v$ newtons.

The mass of the motorcycle and its rider is 500 kg.

(i) Show that $\frac{dv}{dt} = -\frac{v}{25}$.

(2 marks)

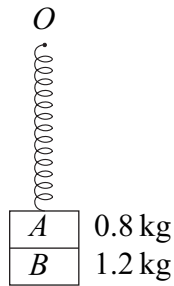
(ii) Hence find the time that it takes for the speed of the motorcycle to reduce from 20 m s^{-1} to 10 m s^{-1} .

(6 marks)

Turn over for the next question

8 Two small blocks, A and B , of masses 0.8 kg and 1.2 kg respectively, are stuck together. A spring has natural length 0.5 metres and modulus of elasticity 49 N . One end of the spring is attached to the top of the block A and the other end of the spring is attached to a fixed point O .

- (a) The system hangs in equilibrium with the blocks stuck together, as shown in the diagram.



Find the extension of the spring.

(3 marks)

- (b) Show that the elastic potential energy of the spring when the system is in equilibrium is 1.96 J . *(2 marks)*
- (c) The system is hanging in this equilibrium position when block B falls off and block A begins to move vertically upwards.

Block A next comes to rest when the spring is **compressed** by x metres.

- (i) Show that x satisfies the equation

$$x^2 + 0.16x - 0.008 = 0 \quad (5\text{ marks})$$

- (ii) Find the value of x . *(2 marks)*

END OF QUESTIONS