

## General Certificate of Education

## Mathematics 6360

MM2B Mechanics 2

# Mark Scheme

## 2006 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

### **Key To Mark Scheme And Abbreviations Used In Marking**

M	mark is for method			
m or dM	mark is dependent on one or more M marks and is for method			
A	mark is dependent on M or m marks and is for accuracy			
В	mark is independent of M or m marks and is for method and accuracy			
Е	mark is for explanation			
$\sqrt{\text{or ft or F}}$	follow through from previous			
	incorrect result	MC	mis-copy	
CAO	correct answer only	MR	mis-read	
CSO	correct solution only	RA	required accuracy	
AWFW	anything which falls within	FW	further work	
AWRT	anything which rounds to	ISW	ignore subsequent work	
ACF	any correct form	FIW	from incorrect work	
AG	answer given	BOD	given benefit of doubt	
SC	special case	WR	work replaced by candidate	
OE	or equivalent	FB	formulae book	
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme	
–x EE	deduct x marks for each error	G	graph	
NMS	no method shown	c	candidate	
PI	possibly implied	sf	significant figure(s)	
SCA	substantially correct approach	dp	decimal place(s)	

#### No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

#### MM2B

Q	Solution	Marks	Total	Comment
1(a)	$KE = \frac{1}{2} \times 0.4 \times 8^2 = 12.8 \text{ J}$	M1		Use of KE formula.
	$\frac{RE \times 0.4 \times 6}{2}$	A1	2	Correct KE
(b)(i)	$KE = 12.8 + 0.4 \times 9.8 \times 6 = 36.32 \text{ J}$ AG	M1		Calculation of GPE
		A1	2	Correct KE from correct expression
				(Allow use of CA equations in solutions)
(ii)	$\frac{1}{2} \times 0.4v^2 = 36.32$ $v = \sqrt{\frac{36.32 \times 2}{0.4}} = 13.5 \text{ ms}^{-1}$	M1		Two term energy equation
	2 30.32	A1		Correct energy equation
	$36.32 \times 2$			
	$v = \sqrt{\frac{0.4}{0.4}} = 13.5 \text{ ms}^{-1}$	A1	3	Correct speed
(iii)	No air resistance			
, ,	No resistance forces			
	Weight is the only force	B1	1	Appropriate assumption
	Total		8	
2(a)	$T\cos 30^\circ = 2 \times 9.8$			
	$_{T}$ 2×9.8 AG	M1		Resolving vertically with two terms
	$T = \frac{2 \times 9.8}{\cos 30^{\circ}}$ AG	A1		Correct equation
	T = 22.6  N	A1	3	Correct <i>T</i> from correct working
	1 – 22.0 10	711	3	Correct From correct working
a x	$v^2$	N/1		Darahina hari-antalla
(b)	$T\cos 60^\circ = 2 \times \frac{v^2}{0.6}$	M1		Resolving horizontally.
		A1		Correct equation
	$v = 1.84 \text{ ms}^{-1}$	dM1		Solving for <i>v</i>
		A1	4	Correct v
	Total		7	
		M1		Differentiating, with at least one term
3(a)(i)	$a = 2 + 12e^{-t}$		2	correct.
(	2 < 14	A1	2	Correct velocity
(ii)	$2 < a \le 14$	B1 B1		For 2 For 14
		B1	3	Correct inequalities
		וע	3	Correct inequalities
(b)	$s - t^2 + 12a^{-t} + c$	M1		Integrating, with at least one term correct
	$s = t^{2} + 12e^{-t} + c$ $s = 0, t = 0 \Rightarrow c = -12$	A1		Correct expression with or without $c$
		dM1		Finding c.
	$s = t^2 + 12e^{-t} - 12$	A1	4	Correct final expression
	Total		9	

#### MM2B cont

Q	Solution	Marks	Total	Comment
4(a)	Because the lamina is symmetrical.	B1	1	Correct explanation
(b)	$\overline{y} = \frac{250 \times 2.5 + 150 \times 7.5}{250 + 150}$	M1		Moment equation with appropriate number of terms
	250+150	<b>A</b> 1		Correct numerator
	$=\frac{1750}{}$			
	400	A1		correct denominator
	$=4.375 \qquad AG$	A1	4	Correct value from correct working
(c)	$\tan \alpha = \frac{10 - 4.375}{25} = \frac{5.625}{25}$	M1 M1		Use of tan. Subtracting from 10
		<b>A</b> 1		Correct expression
	$\alpha = 12.7^{\circ}$	A1	4	Correct angle
(d)	When it has been assumed that the centre of mass of each of the rectangles used is at its centre.  OR			
	Relating area to mass.	B1	1	Correct explanation
	Total		10	
5(a)	$\mathbf{F} = 12\cos t\mathbf{i} - 30\sin t\mathbf{j}$	M1		Use of $\mathbf{F} = m\mathbf{a}$
	$\mathbf{F}(0) = 6 \times 2\mathbf{i} \text{ so } F = 12 \text{ N}$	A1	2	Correct F
		A1	3	Correct magnitude
(b)	$\mathbf{v} = \int 2\cos t dt \mathbf{i} + \int -5\sin t dt \mathbf{j}$	M1		Integrating
	$= (2\sin t + c_1)\mathbf{i} + (5\cos t + c_2)\mathbf{j}$	<b>A</b> 1		Correct i component
		A1		Correct <b>j</b> component Both of above with or without constants
	$\mathbf{v}(0) = 2\mathbf{i} + 10\mathbf{j} \Rightarrow c_1 = 2, c_2 = 5$	dM1		Finding constants of integration
	$\mathbf{v} = (2\sin t + 2)\mathbf{i} + (5\cos t + 5)\mathbf{j}$	A1		Correct final answer
	, , , , , , , , , , , , , , , , , , , ,		5	
	Total		8	

#### MM2B (cont)

Q	Solution	Marks	Total	Comment
6(a)	The boat is not affected by the movement of the water.	В1		First assumption
	The resistance force will directly oppose the motion of the boat and be the only force that needs to be considered.	B1	2	second assumption
	The boat is a particle. / There is no wind. / No air resistance.	B1	1	Appropriate assumption
(c)(i)	$80\frac{dv}{dt} = -20v$ $\frac{dv}{dt} = -\frac{v}{4}$ $\int \frac{1}{v} dv = -\int \frac{1}{4} dt$ $\ln v = -\frac{t}{4} + c$	M1		Use of Newton's second law, $\frac{dv}{dt}$ and 20v.
	$\frac{dv}{dt} = -\frac{v}{4}$	A1	2	Correct result
(ii)	$\int_{V}^{1} dv = -\int_{1}^{1} dt$	M1		Sep. of variables and forming integrals
	$\ln v = -\frac{t}{4} + c$	dM1 A1		Integrating to get an $\ln v$ term Correct integrals with or without $c$
	$v = Ae^{-\frac{t}{4}}$			
	$v = 12, t = 0 \Rightarrow A = 12$	dM1		Finding A or c
	$v = Ae^{-4}$ $v = 12, t = 0 \Rightarrow A = 12$ $v = 12e^{-\frac{t}{4}}$	A1	5	Correct final result
	Total		10	
7(a)	$\frac{1}{2}mv^{2} = \frac{1}{2}m \times 2^{2} + mg(3 - 3\cos\theta)$ $v^{2} = 4 + 6g(1 - \cos\theta)$	M1		Three term energy equation.
	2 2	A1		Correct equation
	$v^2 = 4 + 6g(1 - \cos\theta)$	dM1		Solving for $v^2$
		A1	4	Correct result from correct working
(b)	$mg\cos\theta = m\frac{v^2}{3}$	M1 A1		Resolving towards the centre Correct equation
	$3g\cos\theta = 4 + 6g - 6g\cos\theta$	dM1		Solving for $\cos \theta$
	$\cos\theta = \frac{4+6g}{9g}$	A1		Correct $\cos \theta$
	$\theta = 44.6^{\circ}$	A1	5	Correct angle
	Total		9	

MM2B (cont)

Q	Solution	Marks	Total	Comment
8(a)	$\frac{100}{0.4}e = 10 \times 9.8$ $e = 0.392 \text{ m}$	M1		Use of Hookes law and equilibrium
	Length = $0.392 + 0.4 = 0.792$	<b>A</b> 1	2	Correct length
(b)	$EPE = \frac{1}{2} \times \frac{100}{0.4} \times 0.6^2 = 45 \text{ J} \text{ AG}$	M1 A1	2	Use of EPE formula Correct value from correct working
(c)(i)	$45 = \frac{1}{2} \times \frac{100}{0.4} (x - 0.4)^2 + \frac{1}{2} \times 10v^2 + 10 \times 9.8(1 - x)$	M1 A1		Expression for EPE with $(x \pm 0.4)^2$ Correct EPE
	$45 = 125(x - 0.4)^2 + 5v^2 + 98(1 - x)$	M1		Four term energy equation
	$5v^2 = 98x - 98 + 45 - 125x^2 + 100x - 20$	B1		Correct GPE
	$v^2 = 39.6x - 25x^2 - 14.6$	A1 dM1		Correct equation
	AG	A1	7	Solving for $v^2$ Correct result from correct working
(ii)	$39.6x - 25x^2 - 14.6 = 0$ $25x^2 - 39.6x + 14.6 = 0$	M1		Solving quadratic
	$x = \frac{39.6 \pm \sqrt{39.6^2 - 4 \times 25 \times 14.6}}{2 \times 25}$ = 1 or 0.584	A1		Correct solutions
	x = 0.584	A1 A1	3	Appropriate value selected
				SC Only correct answers given award M1A1.
	Total		14	
	TOTAL		75	