General Certificate of Education
January 2007
Advanced Level Examination

MATHEMATICS
Unit Mechanics 2A

MM2A/W

ASSESSMENT and QUALIFICATIONS ALLIANCE

Tuesday 16 January 2007 9.00 am to 10.15 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2A/W.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 60.
- The marks for questions are shown in brackets.
- Unit Mechanics 2A has a **written paper and coursework**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

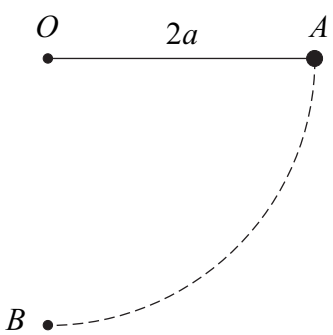
Answer **all** questions.

1 A lift containing miners moves vertically from rest at one level to rest at a higher level.

The total mass of the lift and the miners is 800 kg. The vertical distance between the two levels is 200 metres.

Find the work done in raising the lift and the miners from the lower level to the higher level. *(3 marks)*

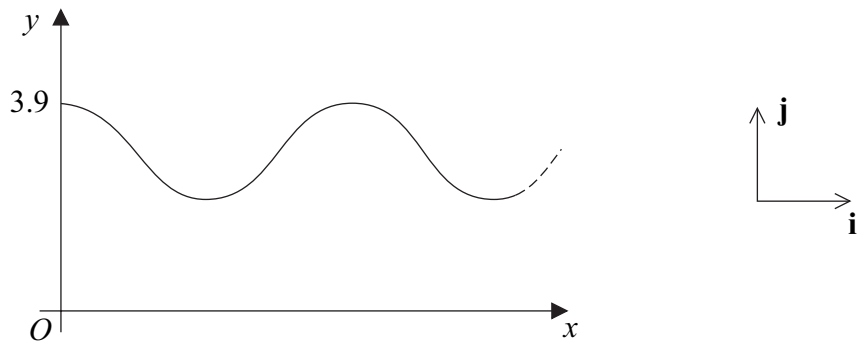
2 A light inextensible string has length $2a$. One end of the string is attached to a fixed point O and a particle of mass m is attached to the other end. Initially, the particle is held at the point A with the string taut and horizontal. The particle is then released from rest and moves in a circular path. Subsequently, it passes through the point B, which is directly below O. The points O, A and B are as shown in the diagram.



(a) Show that the speed of the particle at B is $2\sqrt{ag}$. *(3 marks)*

(b) Find the tension in the string as the particle passes through B. Give your answer in terms of m and g. *(3 marks)*

3 Jane is on a ride in a theme park. Part of the curved path of the ride is shown in the diagram.



Jane's position vector, **r** metres, at time t seconds is given by

$$\mathbf{r} = 1.2t\,\mathbf{i} + (3 + 0.9\cos t)\mathbf{j}$$

where the perpendicular unit vectors **i** and **j** are directed horizontally and vertically upwards respectively.

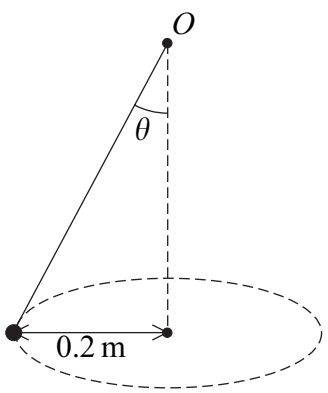
(a) Find an expression for Jane's velocity at time t. *(2 marks)*

(b) (i) Find an expression for Jane's speed at time t. *(2 marks)*

(ii) Find Jane's maximum speed during the ride. *(2 marks)*

Turn over for the next question

4 A particle is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. The particle is set into motion, so that it describes a horizontal circle whose centre is vertically below O. The angle between the string and the vertical is θ, as shown in the diagram.



(a) The particle completes 40 revolutions every minute.

Show that the angular speed of the particle is $\frac{4\pi}{3}$ radians per second. *(2 marks)*

(b) The radius of the circle is 0.2 metres.

Find, in terms of π, the magnitude of the acceleration of the particle. *(2 marks)*

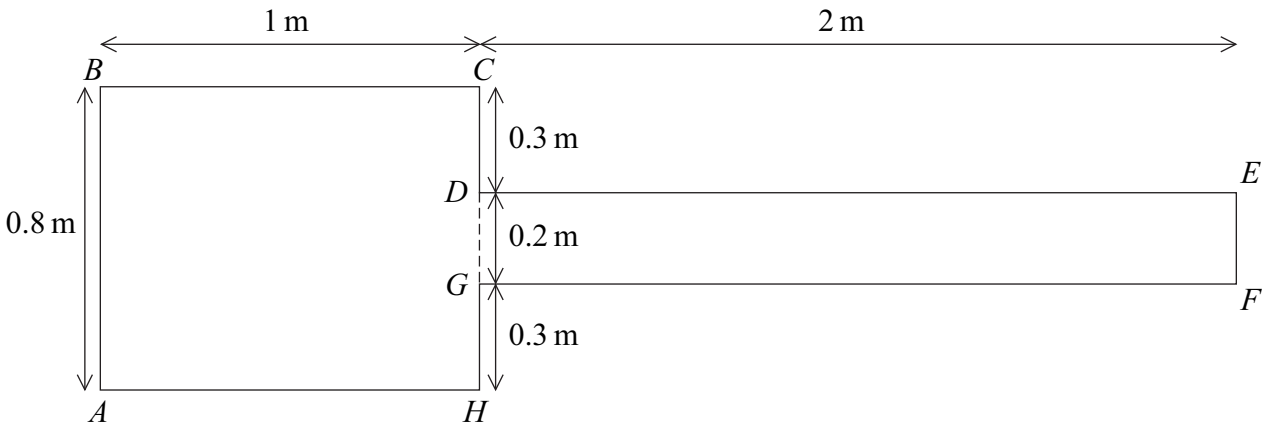
(c) The mass of the particle is m kg and the tension in the string is T newtons.

(i) Draw a diagram showing the forces acting on the particle. *(1 mark)*

(ii) Explain why $T\cos\theta = mg$. *(1 mark)*

(iii) Find the value of θ, giving your answer to the nearest degree. *(5 marks)*

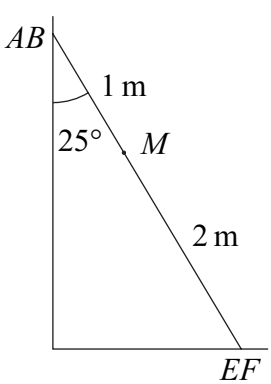
5 A sign advertising a gym consists of two rectangles $ABCH$ and $DEFG$ fixed rigidly together. The sign can be modelled as a uniform lamina, as shown in the diagram.



(a) The centre of mass of the sign is at the point M. Show that M lies on the line CH.

(4 marks)

(b) The sign is placed with its side EF on rough horizontal ground and its side AB against a smooth vertical wall. The sign rests in equilibrium at an angle of 25° with the **vertical**, as shown in the diagram.



The weight of the sign is 90 newtons.

(i) By taking moments, show that the normal reaction force between the sign and the wall is 60 tan 25° newtons. *(3 marks)*

(ii) The coefficient of friction between the sign and the ground is μ.

Show that $\mu \geqslant \frac{2}{3} \tan 25°$. *(4 marks)*

6 A motorcycle has a maximum power of 72 kilowatts. The motorcycle and its rider are travelling along a straight horizontal road. When they are moving at a speed of $V\,\text{m s}^{-1}$, they experience a total resistance force of magnitude kV newtons, where k is a constant.

(a) The maximum speed of the motorcycle and its rider is $60\,\text{m s}^{-1}$.

Show that $k = 20$. *(3 marks)*

(b) When the motorcycle is travelling at $20\,\text{m s}^{-1}$, the rider allows the motorcycle to freewheel so that the only horizontal force acting is the resistance force. When the motorcycle has been freewheeling for t seconds, its speed is $v\,\text{m s}^{-1}$ and the magnitude of the resistance force is $20v$ newtons.

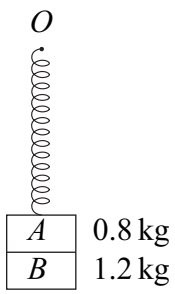
The mass of the motorcycle and its rider is 500 kg.

(i) Show that $\dfrac{\text{d}v}{\text{d}t} = -\dfrac{v}{25}$. *(2 marks)*

(ii) Hence find the time that it takes for the speed of the motorcycle to reduce from $20\,\text{m s}^{-1}$ to $10\,\text{m s}^{-1}$. *(6 marks)*

7 Two small blocks, A and B, of masses 0.8 kg and 1.2 kg respectively, are stuck together. A spring has natural length 0.5 metres and modulus of elasticity 49 N. One end of the spring is attached to the top of the block A and the other end of the spring is attached to a fixed point O.

(a) The system hangs in equilibrium with the blocks stuck together, as shown in the diagram.



Find the extension of the spring. *(3 marks)*

(b) Show that the elastic potential energy of the spring when the system is in equilibrium is 1.96 J. *(2 marks)*

(c) The system is hanging in this equilibrium position when block B falls off and block A begins to move vertically upwards.

Block A next comes to rest when the spring is **compressed** by x metres.

(i) Show that x satisfies the equation

$$x^2 + 0.16x - 0.008 = 0$$ *(5 marks)*

(ii) Find the value of x. *(2 marks)*

END OF QUESTIONS

There are no questions printed on this page