

General Certificate of Education
June 2007
Advanced Level Examination



MATHEMATICS
Unit Mechanics 5

MM05

Tuesday 26 June 2007 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM05.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A particle moves with simple harmonic motion along a straight line. Its maximum speed is 4 m s^{-1} and its maximum acceleration is 100 m s^{-2} .

(a) Show that the period of motion is $\frac{2\pi}{25}$ seconds. *(4 marks)*

(b) Find the amplitude of the motion. *(1 mark)*

2 A simple pendulum consists of a particle, of mass m , fixed to one end of a light, inextensible string of length l . The other end of the string is attached to a fixed point. When the pendulum is in motion, the angle between the string and the downward vertical is θ at time t . The motion takes place in a vertical plane.

(a) Show, using a trigonometrical approximation, that for small angles of oscillation the motion of the pendulum can be modelled by the differential equation

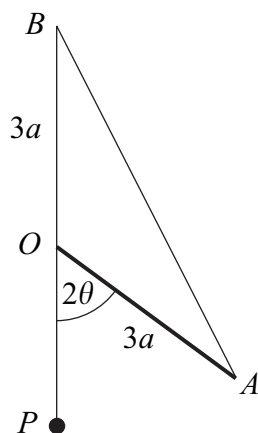
$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta \quad (4 \text{ marks})$$

(b) The pendulum has length 0.5 metres. The pendulum is released from rest with the string taut and at an angle of $\frac{\pi}{400}$ to the vertical.

(i) Given that $\theta = A \cos \omega t$, find the values of A and ω . *(3 marks)*

(ii) Find the maximum speed of the particle in the subsequent motion. *(3 marks)*

- 3 A uniform rod, OA , of length $3a$ and mass $2m$, is freely pivoted at O . A light, inextensible string, of length $10a$, is attached to the rod at A and passes over a fixed, smooth peg at B , a distance $3a$ vertically above O . A particle, P , of mass m , is attached to the other end of the string. The angle between the rod and the vertical is 2θ , as shown in the diagram.



- (a) Show that the total potential energy of the system, V , is given by

$$V = 6mga \cos \theta - 7mga - 3mga \cos 2\theta$$

where gravitational potential energy is taken to be zero at O . (5 marks)

- (b) Find the **two** values of θ , $0 \leq \theta < \frac{\pi}{2}$, for which the system is in equilibrium. (6 marks)

- (c) Determine the stability of each position of equilibrium. (4 marks)

- 4 A particle of mass m is moving along a smooth wire that is fixed in a plane. The polar equation of the wire is $r = ae^{3\theta}$. The particle moves with a constant angular velocity of 6.

At time $t = 0$, the particle is at the point with polar coordinates $(a, 0)$.

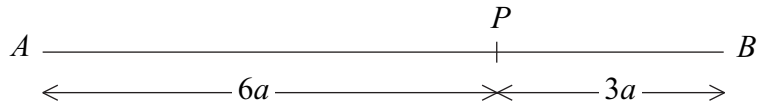
- (a) Find the transverse and radial components of the acceleration of the particle in terms of a and t . (10 marks)

- (b) The resultant force on the particle is \mathbf{F} .

Show that the magnitude of \mathbf{F} , at time t , is $360mae^{18t}$. (4 marks)

- 5 The ends of a light, uniform elastic string are fixed to two points, A and B , a distance $9a$ apart on a smooth, horizontal plane. The string is of natural length $6a$ and modulus of elasticity $4mn^2a$, where n is a constant.

A particle of mass m is attached to the string at P , where $AP = 6a$. The natural length of AP is $4a$ and the natural length of BP is $2a$. In this position, the particle is in equilibrium.



The particle is moved a distance $\frac{1}{2}a$ towards B and then released from rest at time $t = 0$. The displacement of the particle from its equilibrium position at time t is x . Hence initially $x = +\frac{1}{2}a$.

The motion of the particle is resisted by a force of magnitude $2mnv$, where v is the speed of the particle at time t .

- (a) Show that x satisfies

$$\frac{d^2x}{dt^2} + 2n \frac{dx}{dt} + 3n^2x = 0 \quad (7 \text{ marks})$$

- (b) Given that $n = 1$, find x in terms of a and t . (8 marks)

- 6 A large snowball, which may be modelled as a uniform sphere of radius r , moves with speed v down a slope inclined at 30° to the horizontal. The snowball picks up snow at a rate proportional to both its speed and its mass, m , and hence it may be assumed that $\frac{dm}{dt} = kmv$ at time t , where k is a constant.

You should ignore any rotational motion of the snowball.

- (a) Neglecting any resistance forces acting on the snowball, show that

$$2\frac{dv}{dt} + 2kv^2 = g \quad (4 \text{ marks})$$

- (b) Using the identity

$$\frac{dv}{dt} = \frac{dx}{dt} \times \frac{dv}{dx} = v \frac{dv}{dx}$$

where x is the distance travelled by the centre of the snowball, show that the differential equation in part (a) can be written as

$$2v \frac{dv}{dx} = g - 2kv^2 \quad (1 \text{ mark})$$

- (c) At time $t = 0$, $v = 0$ and $x = 0$.

Solve the differential equation in part (b) to find v^2 as a function of x . (6 marks)

- (d) When $t = 0$, $v = 0$ and $x = 0$, the radius of the snowball is $\frac{1}{3}$ metre.

(i) Show that $r^3 = Ce^{kx}$, where C is a constant to be determined. (3 marks)

(ii) Find, in terms of g and k , the speed of the snowball when its radius is 1 metre. (2 marks)

END OF QUESTIONS

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