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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MM05 Mechanics 5

Mark Scheme

2007 examination - June series

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Key to mark scheme and abbreviations used in marking

	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM05

Q	Solution	Marks	Total	Comments
1(a)	Maximum speed $\Rightarrow \omega a = 4$	B1		
	Maximum acceleration $\Rightarrow \omega^2 a = 100$	B1		
	$\omega = 25$	M1		
	Period is $\frac{2\pi}{\omega}$			
	$= \frac{2\pi}{25}$	A1	4	AG; needs to use a justified $\omega = 25$
(b)	Amplitude is $\frac{4}{25}$ m	B1	1	
	Total		**5**	
2(a)	Using transverse component of acceleration is $r\frac{d^2\theta}{dt^2}$	B1		
	$ml\frac{d^2\theta}{dt^2} = -mg\sin\theta$	M1		
	$\frac{d^2\theta}{dt^2} = -\frac{g\sin\theta}{l}$			
	For small angles of θ, $\sin\theta \approx \theta$	B1		
	$\frac{d^2\theta}{dt^2} = -\frac{g\theta}{l}$	A1	4	AG
(b)(i)	$A = \frac{\pi}{400}$	B1		
	$\omega = \sqrt{\frac{g}{l}}$	M1		
	$= \sqrt{\frac{9.8}{0.5}} = \frac{7\sqrt{10}}{5}$ or 4.43	A1	3	
(ii)	Maximum speed is $a\omega$			
	$= \frac{7\sqrt{10}}{5} \times 0.5 \times \frac{\pi}{400}$	M1A1		Needs 0.5 term
	= 0.0174	A1	3	$\sqrt{\frac{g}{2}} \times \frac{\pi}{400}$
	Total		**10**	

MM05 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$AB = 6a\cos\theta$	M1A1		
	Potential energy, below O, of rod is $-2mga\frac{3}{2}\cos 2\theta = -3mga\cos 2\theta$	B1		
	Potential energy, below O, of particle is $-mg(7a - 6a\cos\theta)$	B1		
	$= 6mga\cos\theta - 7mga$			
	$V = 6mga\cos\theta - 7mga - 3mga\cos 2\theta$	A1	5	AG
(b)	At equilibrium, $\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0$	M1		
	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = 6mga\sin 2\theta - 6mga\sin\theta$	M1A1		
	$= 6mga\sin\theta(2\cos\theta - 1)$			
	$= 0$ when			
	$\sin\theta = 0$ or $\cos\theta = \frac{1}{2}$	A1		
	$\therefore$ system is in equilibrium when			
	$\theta = 0$ and $\frac{\pi}{3}$	A1,A1	6	
(c)	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = 12mga\cos 2\theta - 6mga\cos\theta$	M1		
	When $\theta = 0$, $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = 6mga$	A1		
	This is positive $\Rightarrow$ minimum PE Position is stable equilibrium	E1		
	When $\theta = \frac{\pi}{3}$, $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -9mga$			
	$\Rightarrow$ maximum PE Position is unstable equilibrium	E1	4	
	Total		**15**	

MM05 (cont)

Q		Marks	Total	Comments
4(a)	$r = ae^{3\theta}$			
	$\dot{r} = 3ae^{3\theta}\dot{\theta}$	M1		
	$\ddot{r} = 9ae^{3\theta}\dot{\theta}^2$	M1		
	Since $\ddot{\theta} = 0$,	B1		B1 for $\ddot{\theta} = 0$
	$\dot{r} = 18ae^{3\theta}$	A1		
	$\ddot{r} = 324ae^{3\theta}$	A1		
	Since $\dot{\theta}$ is a constant, $\theta = 6t$ and $\theta = 0$ when $t = 0$	B1		
	Transverse acceleration is $2\dot{r}\dot{\theta} + r\ddot{\theta}$	M1		
	$= 216ae^{18t}$	A1		
	Radial acceleration is $\ddot{r} - r\dot{\theta}^2$			
	$= 324ae^{18t} - 36ae^{18t}$	M1		
	$= 288ae^{18t}$	A1	10	
(b)	Using $F = ma$, $\mathbf{F} = 288mae^{18t}\hat{r} + 216mae^{18t}\hat{\theta}$	M1A1		
	Magnitude is $\{(288mae^{18t})^2 + (216mae^{18t})^2\}^{1/2}$	M1		
	$= 360mae^{18t}$	A1	4	AG
	Total		**14**	

MM05 (cont)

Q	Solution	Marks	Total	Comments
5(a)	Natural length of AP is $4a$ and natural length of BP is $2a$			
	When particle is x from equilibrium position:			
	Tension in AP is $\dfrac{4mn^2a(2a+x)}{4a}$	M1A1		
	Tension in BP is $\dfrac{4mn^2a(a-x)}{2a}$	M1A1		
	In general position, using $F = ma$: $m\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = \dfrac{4mn^2a(a-x)}{2a} - \dfrac{4mn^2a(2a+x)}{4a} - 2mn\dfrac{\mathrm{d}x}{\mathrm{d}t}$	M1A1		
	$m\ddot{x} =$ $2mn^2a - 2mn^2x - 2mn^2a - mn^2x - 2mn\dot{x}$			
	$\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2n\dfrac{\mathrm{d}x}{\mathrm{d}t} + 3n^2x = 0$	A1	7	AG
(b)	[Substituting $x = Ae^{pt}$]			
	$p^2 + 2p + 3 = 0$	M1A1		
	$p = -1 \pm \sqrt{2}\mathrm{i}$	A1		
	General solution is: $x = e^{-t}(A\cos\sqrt{2}t + B\sin\sqrt{2}t)$	A1		
	When $t = 0$, $x = \frac{1}{2}a \Rightarrow \frac{1}{2}a = A$	B1		
	Differentiating: $\dfrac{\mathrm{d}x}{\mathrm{d}t} = -e^{-t}(A\cos\sqrt{2}t + B\sin\sqrt{2}t) +$ $e^{-t}(-A\sqrt{2}\sin\sqrt{2}t + B\sqrt{2}\cos\sqrt{2}t)$	M1A1		
	When $t = 0$, $\dfrac{\mathrm{d}x}{\mathrm{d}t} = 0$			
	$\Rightarrow 0 = -A + \sqrt{2}B$			
	$A = \frac{1}{2}a$, $B = \dfrac{1}{2\sqrt{2}}a$	A1	8	
	$x = ae^{-t}(\frac{1}{2}\cos\sqrt{2}t + \dfrac{1}{2\sqrt{2}}\sin\sqrt{2}t)$			
	Total		**15**	

MM05 (cont)

Q	Solution	Marks	Total	Comments
6(a)	Change in linear momentum = work done by external force			
	$(m + \delta m)(v + \delta v) - mv = mg \sin 30\ \delta t$	M1A1		Needs δ terms
	$v\delta m + m\delta v = \frac{1}{2} mg\delta t$ (to first order of δ terms)			
	$\frac{1}{2} mg = m\frac{\mathrm{d}v}{\mathrm{d}t} + v\frac{\mathrm{d}m}{\mathrm{d}t}$	M1		Accept $mg\sin 30 = \frac{1}{2}mg = \frac{m\mathrm{d}v}{\mathrm{d}t} + kmv^2$
	Using $\frac{\mathrm{d}m}{\mathrm{d}t} = kmv$:			
	$m\frac{\mathrm{d}v}{\mathrm{d}t} + v\,kmv = \frac{1}{2}mg$			
	$2\frac{\mathrm{d}v}{\mathrm{d}t} + 2kv^2 = g$	A1	4	AG
(b)	Using the identity $\frac{\mathrm{d}v}{\mathrm{d}t} = v\frac{\mathrm{d}v}{\mathrm{d}x}$:			
	$2v\frac{\mathrm{d}v}{\mathrm{d}x} + 2kv^2 = g$			
	$2v\frac{\mathrm{d}v}{\mathrm{d}x} = g - 2kv^2$	B1	1	AG
(c)	$\int \frac{2v}{g - 2kv^2}\,\mathrm{d}v = \int \mathrm{d}x$	M1		
	$-\frac{1}{2k}\ln(g - 2kv^2) = x + c$	A1		
	When $x = 0$, $v = 0 \Rightarrow c = -\frac{1}{2k}\ln g$	M1		
	$x = \frac{1}{2k}\ln\frac{g}{g - 2kv^2}$	M1A1		
	$\frac{g}{g - 2kv^2} = \mathrm{e}^{2kx}$			
	$g\mathrm{e}^{-2kx} = g - 2kv^2$			
	$v^2 = \frac{g(1 - \mathrm{e}^{-2kx})}{2k}$	A1	6	

MM05 (cont)

Q	Solution	Marks	Total	Comments
6(d)(i)	Using $m = \frac{4}{3}\pi r^3 \rho$: $\frac{dm}{dt} = kmv \Rightarrow$ $4\pi r^2 \rho \frac{dr}{dt} = k\frac{4}{3}\pi r^3 \rho v$ $3\frac{dr}{dt} = krv$	M1		
	$3\int \frac{dr}{r} = \int kv\ dt$ $= \int k\ dx$ $3\ln r = kx + c$ $r^3 = Ce^{kx}$ When $x = 0, r = \frac{1}{3} \Rightarrow C = \frac{1}{27}$	A1		
	$r^3 = \frac{1}{27}e^{kx}$	B1	3	
(ii)	When $r = 1$, $e^{kx} = 27$ Using result in (c), $v^2 = \frac{g(1-\frac{1}{729})}{2k}$	M1		
	$v = \sqrt{\frac{364}{729}\frac{g}{k}}$	A1	2	
	Total		**16**	
	TOTAL		**75**	