

General Certificate of Education
June 2007
Advanced Level Examination



MATHEMATICS
Unit Mechanics 4

MM04

Monday 18 June 2007 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
 - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM04.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

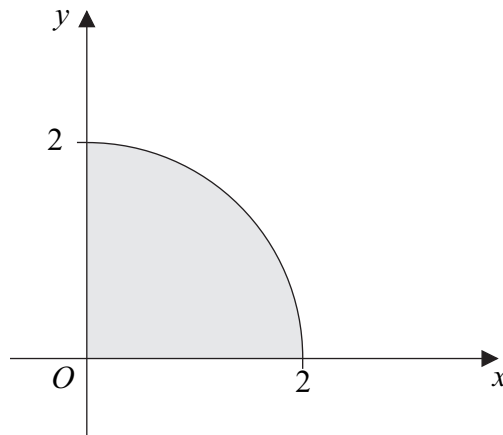
- 1 Two forces, $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$, act at the points $P(1, -1, 6)$ and $Q(0, 3, -2)$ respectively. These two forces together with a force \mathbf{F} , acting through the point Q , form a couple.

(a) (i) Find the force \mathbf{F} . (3 marks)

(ii) Show that the magnitude of \mathbf{F} is $3\sqrt{10}$. (2 marks)

(b) Find the moment of the couple. (5 marks)

- 2 The region bounded by the positive x -axis, the positive y -axis and the curve with equation $y = \sqrt{4 - x^2}$ is shown in the diagram.



The region is rotated through 2π radians around the x -axis to form a uniform solid.

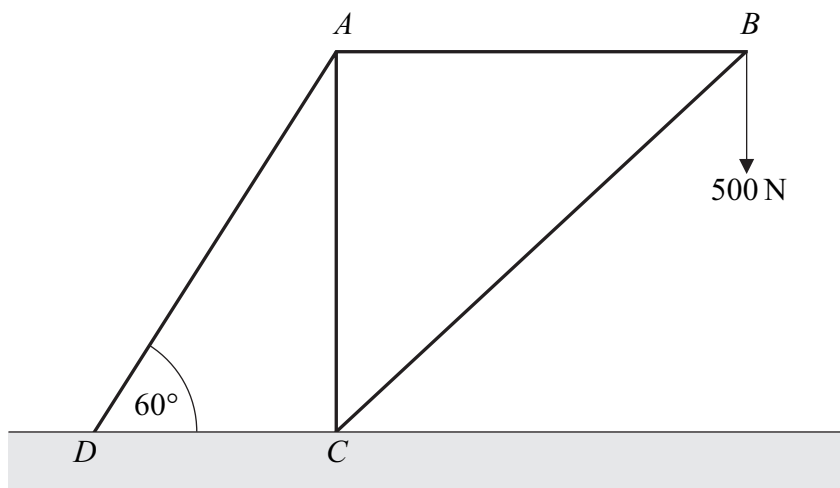
(a) Use integration to show that the volume of the solid is $\frac{16\pi}{3}$. (3 marks)

(b) Use integration to find the distance of the centre of mass of the solid from the y -axis. (4 marks)

(c) The solid is suspended from a point on the edge of its circular face and hangs in equilibrium.

Find the angle between the circular face and the vertical. (3 marks)

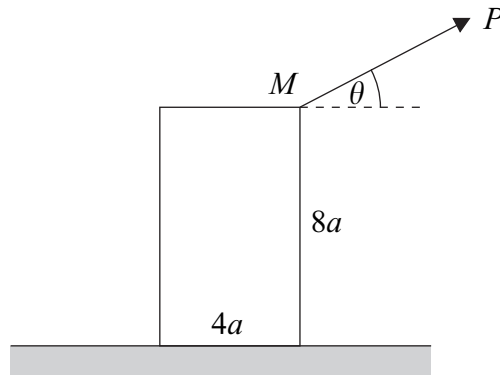
- 3 Dominic designs a crane. He models the crane by four light smoothly-jointed rods AB , AC , AD and BC . The crane is smoothly jointed to the horizontal ground at C and D . The framework is at rest in a vertical plane, with AB horizontal and AC vertical. The framework is such that $AC = AB$ and the angle CDA is 60° . A force of magnitude 500 N acts vertically downwards on the framework at B , as shown in the diagram.



- (a) (i) Find the magnitudes of the forces in the rods BC , AB and AD . (9 marks)
- (ii) State which of the rods BC , AB and AD could be replaced by ropes, giving a reason for your answer. (3 marks)
- (b) Find the magnitude of the force acting on the framework at D . (1 mark)

Turn over for the next question

- 4 A uniform block in the shape of a cuboid has weight W , a square base of side $4a$, a height of $8a$ and stands on a rough horizontal surface. The coefficient of friction between the block and the surface is μ . A rope is attached to the point M , the midpoint of a top edge of the block.



The rope is pulled with a force P , which acts at an angle of θ above the horizontal and is perpendicular to the top edge.

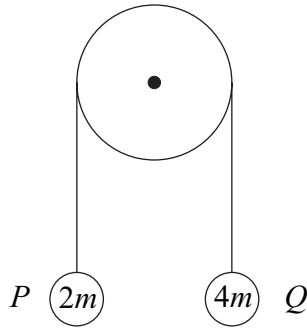
- (a) Find P , in terms of W and θ , if the block is on the point of toppling. (4 marks)
- (b) Show that $P = \frac{\mu W}{\cos \theta + \mu \sin \theta}$ if the block is on the point of sliding. (7 marks)
- (c) Given that $\tan \theta = 1$, find an inequality that μ must satisfy if the block slides before it topples. (5 marks)
- 5 A thin uniform rectangular plate, of mass m and with sides of length $4a$ and $6a$, is hanging freely from a horizontal axis along a side of length $4a$. Initially the plate is stationary and hangs in a vertical plane.

- (a) Use integration to show that the moment of inertia of the plate about the horizontal axis is $12ma^2$. (5 marks)
- (b) The plate is struck at its centre of mass by a small lump of clay of mass $\frac{1}{2}m$ moving with speed u horizontally at right angles to the plate.

Assume that the clay sticks to the plate throughout the subsequent motion. Find, in terms of u and a , the angular speed of the plate immediately after impact. (7 marks)

- 6 A uniform circular disc, of radius a , can rotate freely in a vertical plane about a fixed horizontal axis through its centre perpendicular to its plane face. The moment of inertia of the disc about this axis is $10ma^2$.

A light inextensible string passes over the rough rim of the disc, and two particles P and Q , of masses $2m$ and $4m$ respectively, are attached to its ends.



Initially the system is held at rest with the particles hanging freely in equilibrium. The system is then released. In the subsequent motion, no slipping occurs between the string and the disc. When the disc has turned through an angle θ , the particle P has **not** reached the disc.

- (a) (i) Show that the gain in kinetic energy of the system is $8ma^2\dot{\theta}^2$. (4 marks)
- (ii) Hence show that $a\dot{\theta}^2 = \frac{1}{4}g\theta$. (3 marks)
- (b) Find the force exerted by the string:
- (i) on the particle P ;
- (ii) on the particle Q . (7 marks)

END OF QUESTIONS

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