

General Certificate of Education
June 2007
Advanced Level Examination



MATHEMATICS
Unit Mechanics 3

MM03

Monday 11 June 2007 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM03.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 The magnitude of the gravitational force, F , between two planets of masses m_1 and m_2 with centres at a distance x apart is given by

$$F = \frac{Gm_1m_2}{x^2}$$

where G is a constant.

- (a) By using dimensional analysis, find the dimensions of G . (3 marks)
- (b) The lifetime, t , of a planet is thought to depend on its mass, m , its initial radius, R , the constant G and a dimensionless constant, k , so that

$$t = km^\alpha R^\beta G^\gamma$$

where α , β and γ are constants.

Find the values of α , β and γ . (5 marks)

- 2 The unit vectors \mathbf{i} , \mathbf{j} and \mathbf{k} are directed due east, due north and vertically upwards respectively.

Two helicopters, A and B , are flying with constant velocities of $(20\mathbf{i} - 10\mathbf{j} + 20\mathbf{k}) \text{ m s}^{-1}$ and $(30\mathbf{i} + 10\mathbf{j} + 10\mathbf{k}) \text{ m s}^{-1}$ respectively. At noon, the position vectors of A and B relative to a fixed origin, O , are $(8000\mathbf{i} + 1500\mathbf{j} + 3000\mathbf{k}) \text{ m}$ and $(2000\mathbf{i} + 500\mathbf{j} + 1000\mathbf{k}) \text{ m}$ respectively.

- (a) Write down the velocity of A relative to B . (2 marks)
- (b) Find the position vector of A relative to B at time t seconds after noon. (3 marks)
- (c) Find the value of t when A and B are closest together. (5 marks)

- 3 A particle P , of mass 2 kg, is initially at rest at a point O on a smooth horizontal surface. The particle moves along a straight line, OA , under the action of a horizontal force. When the force has been acting for t seconds, it has magnitude $(4t + 5) \text{ N}$.

- (a) Find the magnitude of the impulse exerted by the force on P between the times $t = 0$ and $t = 3$. (3 marks)
- (b) Find the speed of P when $t = 3$. (2 marks)
- (c) The speed of P at A is 37.5 m s^{-1} . Find the time taken for the particle to reach A . (4 marks)

4 Two small smooth spheres, A and B , of equal radii have masses 0.3 kg and 0.2 kg respectively. They are moving on a smooth horizontal surface directly towards each other with speeds 3 m s^{-1} and 2 m s^{-1} respectively when they collide. The coefficient of restitution between A and B is 0.8 .

(a) Find the speeds of A and B immediately after the collision. (6 marks)

(b) Subsequently, B collides with a fixed smooth vertical wall which is at right angles to the path of the sphere. The coefficient of restitution between B and the wall is 0.7 .

Show that B will collide again with A . (3 marks)

5 A ball is projected with speed $u \text{ m s}^{-1}$ at an angle of elevation α above the horizontal so as to hit a point P on a wall. The ball travels in a vertical plane through the point of projection. During the motion, the horizontal and upward vertical displacements of the ball from the point of projection are x metres and y metres respectively.

(a) Show that, during the flight, the equation of the trajectory of the ball is given by

$$y = x \tan \alpha - \frac{gx^2}{2u^2} (1 + \tan^2 \alpha) \quad (6 \text{ marks})$$

(b) The ball is projected from a point 1 metre vertically below and R metres horizontally from the point P .

(i) By taking $g = 10 \text{ m s}^{-2}$, show that R satisfies the equation

$$5R^2 \tan^2 \alpha - u^2 R \tan \alpha + 5R^2 + u^2 = 0 \quad (2 \text{ marks})$$

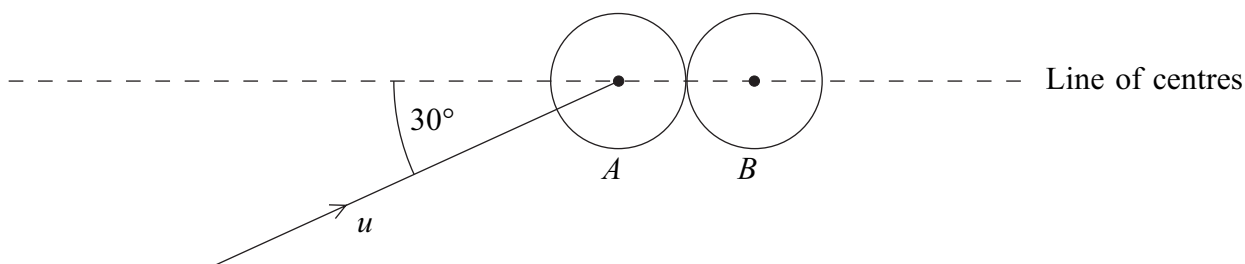
(ii) Hence, given that u and R are constants, show that, for $\tan \alpha$ to have real values, R must satisfy the inequality

$$R^2 \leq \frac{u^2(u^2 - 20)}{100} \quad (2 \text{ marks})$$

(iii) Given that $R = 5$, determine the minimum possible speed of projection. (3 marks)

- 6 A smooth spherical ball, A , is moving with speed u in a straight line on a smooth horizontal table when it hits an identical ball, B , which is at rest on the table.

Just before the collision, the direction of motion of A makes an angle of 30° with the line of the centres of the two balls, as shown in the diagram.



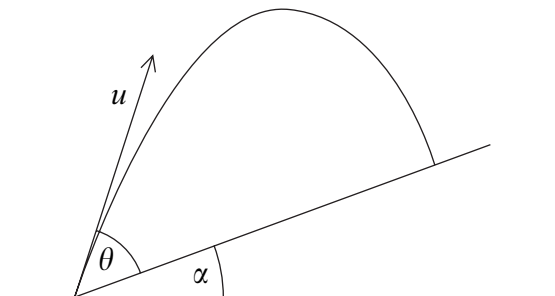
The coefficient of restitution between A and B is e .

- (a) Given that $\cos 30^\circ = \frac{\sqrt{3}}{2}$, show that the speed of B immediately after the collision is

$$\frac{\sqrt{3}}{4}u(1 + e) \quad (5 \text{ marks})$$

- (b) Find, in terms of u and e , the components of the velocity of A , parallel and perpendicular to the line of centres, immediately after the collision. (3 marks)
- (c) Given that $e = \frac{2}{3}$, find the angle that the velocity of A makes with the line of centres immediately after the collision. Give your answer to the nearest degree. (3 marks)

- 7 A particle is projected from a point on a plane which is inclined at an angle α to the horizontal. The particle is projected up the plane with velocity u at an angle θ above the plane. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



- (a) Using the identity $\cos(A + B) = \cos A \cos B - \sin A \sin B$, show that the range up the plane is

$$\frac{2u^2 \sin \theta \cos(\theta + \alpha)}{g \cos^2 \alpha} \quad (8 \text{ marks})$$

- (b) Hence, using the identity $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$, show that, as θ varies, the range up the plane is a maximum when $\theta = \frac{\pi}{4} - \frac{\alpha}{2}$. (3 marks)

- (c) Given that the particle strikes the plane at right angles, show that

$$2 \tan \theta = \cot \alpha \quad (4 \text{ marks})$$

END OF QUESTIONS

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