General Certificate of Education
June 2006
Advanced Level Examination

MATHEMATICS
Unit Mechanics 3

MM03

Wednesday 21 June 2006 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM03.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The time T taken for a simple pendulum to make a single small oscillation is thought to depend only on its length l, its mass m and the acceleration due to gravity g.

By using dimensional analysis:

(a) show that T does **not** depend on m; *(3 marks)*

(b) express T in terms of l, g and k, where k is a dimensionless constant. *(4 marks)*

2 Three smooth spheres A, B and C of equal radii and masses m, m and $2m$ respectively lie at rest on a smooth horizontal table. The centres of the spheres lie in a straight line with B between A and C. The coefficient of restitution between any two spheres is e.

The sphere A is projected directly towards B with speed u and collides with B.

(a) Find, in terms of u and e, the speed of B immediately after the impact between A and B. *(5 marks)*

(b) The sphere B subsequently collides with C. The speed of C immediately after this collision is $\frac{3}{8}u$. Find the value of e. *(7 marks)*

3 A ball of mass 0.45 kg is travelling horizontally with speed $15\,\mathrm{m\,s^{-1}}$ when it strikes a fixed vertical bat directly and rebounds from it. The ball stays in contact with the bat for 0.1 seconds.

At time t seconds after first coming into contact with the bat, the force exerted on the ball by the bat is $1.4 \times 10^5(t^2 - 10t^3)$ newtons, where $0 \leqslant t \leqslant 0.1$.

In this simple model, ignore the weight of the ball and model the ball as a particle.

(a) Show that the magnitude of the impulse exerted by the bat on the ball is 11.7 N s, correct to three significant figures. *(4 marks)*

(b) Find, to two significant figures, the speed of the ball immediately after the impact. *(4 marks)*

(c) Give a reason why the speed of the ball immediately after the impact is different from the speed of the ball immediately before the impact. *(1 mark)*

4 The unit vectors **i** and **j** are directed due east and due north respectively.

Two cyclists, Aazar and Ben, are cycling on straight horizontal roads with constant velocities of $(6\mathbf{i} + 12\mathbf{j})\,\text{km h}^{-1}$ and $(12\mathbf{i} - 8\mathbf{j})\,\text{km h}^{-1}$ respectively. Initially, Aazar and Ben have position vectors $(5\mathbf{i} - \mathbf{j})$ km and $(18\mathbf{i} + 5\mathbf{j})$ km respectively, relative to a fixed origin.

(a) Find, as a vector in terms of **i** and **j**, the velocity of Ben relative to Aazar. *(2 marks)*

(b) The position vector of Ben relative to Aazar at time t hours after they start is **r** km.

Show that

$$\mathbf{r} = (13 + 6t)\mathbf{i} + (6 - 20t)\mathbf{j}$$ *(4 marks)*

(c) Find the value of t when Aazar and Ben are closest together. *(6 marks)*

(d) Find the closest distance between Aazar and Ben. *(2 marks)*

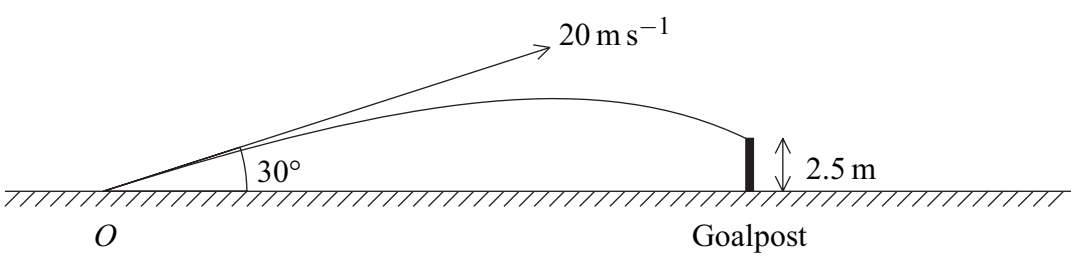
5 A football is kicked from a point O on a horizontal football ground with a velocity of $20\,\text{m s}^{-1}$ at an angle of elevation of 30°. During the motion, the horizontal and upward vertical displacements of the football from O are x metres and y metres respectively.

(a) Show that x and y satisfy the equation

$$y = x\tan 30° - \frac{gx^2}{800\cos^2 30°}$$ *(6 marks)*

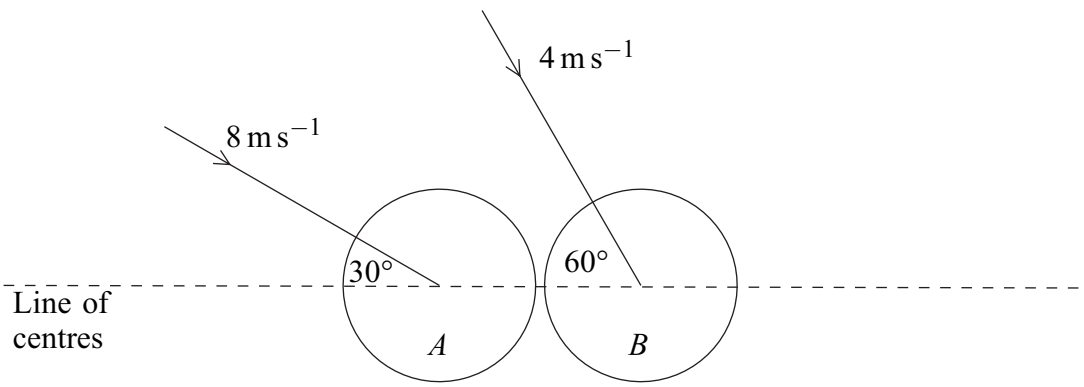
(b) On its downward flight the ball hits the horizontal crossbar of the goal at a point which is 2.5 m above the ground. Using the equation given in part (a), find the horizontal distance from O to the goal.

(4 marks)



(c) State **two** modelling assumptions that you have made. *(2 marks)*

6 Two smooth billiard balls A and B, of identical size and equal mass, move towards each other on a horizontal surface and collide. Just before the collision, A has velocity $8\,\mathrm{m\,s^{-1}}$ in a direction inclined at 30° to the line of centres of the balls, and B has velocity $4\,\mathrm{m\,s^{-1}}$ in a direction inclined at 60° to the line of centres, as shown in the diagram.



The coefficient of restitution between the balls is $\frac{1}{2}$.

(a) Find the speed of B immediately after the collision. *(9 marks)*

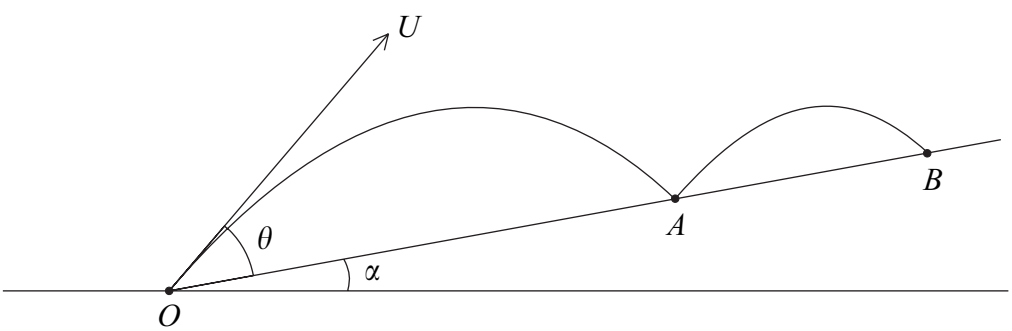
(b) Find the angle between the velocity of B and the line of centres of the balls immediately after the collision. *(2 marks)*

7 A projectile is fired from a point O on the slope of a hill which is inclined at an angle α to the horizontal. The projectile is fired up the hill with velocity U at an angle θ above the hill and first strikes it at a point A. The projectile is modelled as a particle and the hill is modelled as a plane with OA as a line of greatest slope.

(a) (i) Find, in terms of U, g, α and θ, the time taken by the projectile to travel from O to A. *(3 marks)*

(ii) Hence, or otherwise, show that the magnitude of the component of the velocity of the projectile perpendicular to the hill, when it strikes the hill at the point A, is the same as it was initially at O. *(3 marks)*

(b) The projectile rebounds and strikes the hill again at a point B. The hill is smooth and the coefficient of restitution between the projectile and the hill is e.



Find the ratio of the time of flight from O to A to the time of flight from A to B. Give your answer in its simplest form. *(4 marks)*

END OF QUESTIONS

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