General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS
Unit Mechanics 3

MM03

Tuesday 28 June 2005 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM03.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 An expression for calculating the volume per second of liquid flowing through a cylindrical pipe is of the form

$$k l^{-1} r^a p^b \eta^c$$

where k is a dimensionless constant;
l metres is the length of the pipe;
r metres is the radius of the pipe;
p is the excess pressure, measured in $\mathrm{N\,m^{-2}}$;
and η is the viscosity of the liquid measured in $\mathrm{kg\,m^{-1}s^{-1}}$.

(a) (i) Write down the dimensions of η. *(1 mark)*

(ii) Determine the dimensions of p. *(2 marks)*

(b) By using dimensional analysis, find the values of a, b and c. *(6 marks)*

2 The unit vectors **i** and **j** are directed due east and due north respectively.

At 1:00 am, two ships, A and B, have position vectors $(3\mathbf{i}+2\mathbf{j})$ km and $(-4\mathbf{i}+7\mathbf{j})$ km respectively, relative to an origin at a lighthouse O. The ships, A and B, are moving with constant velocities of $(-5\mathbf{i}+8\mathbf{j})\,\mathrm{km\,h^{-1}}$ and $(2\mathbf{i}+3\mathbf{j})\,\mathrm{km\,h^{-1}}$ respectively.

(a) Write down the position vectors of A and B, relative to O, at time t hours after 1:00 am. *(2 marks)*

(b) Show that A and B will collide at 2:00 am. *(2 marks)*

(c) In order to prevent a collision, ship B changes its velocity to $(2\mathbf{i}+10\mathbf{j})\,\mathrm{km\,h^{-1}}$ at 1:45 am. Ship A does not change its velocity.

(i) Show that, at time T hours after 1:45 am, the position vector of B relative to A is given by $(7T-1.75)\mathbf{i}+(2T+1.25)\mathbf{j}$ km. *(6 marks)*

(ii) Find the distance between A and B at 2:00 am. *(2 marks)*

3 Three smooth spheres, A, B and C, of equal radii lie at rest in a straight line on a smooth horizontal table. Sphere A has mass $4m$ and is set in motion with velocity u. It collides directly with sphere B which has mass $2m$. The coefficient of restitution between A and B is e.

(a) Find, in terms of e and u, the velocities of A and B immediately after the collision. *(5 marks)*

(b) Given that the magnitude of the impulse exerted by A on B is $\frac{12mu}{5}$, show that

$$e = \frac{4}{5}$$ *(3 marks)*

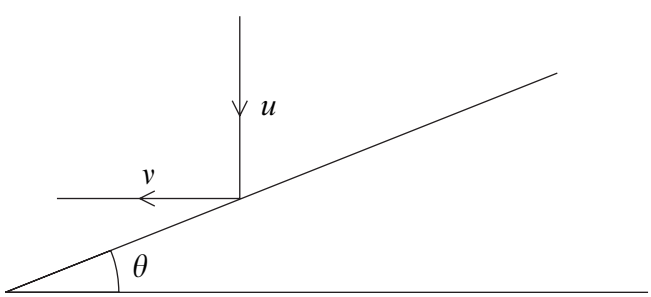
(c) After the collision between A and B, the sphere B collides directly with the sphere C which has mass m. The coefficient of restitution between B and C is $\frac{4}{5}$.

Find, in terms of u, the velocity of B after this collision. *(4 marks)*

(d) Determine whether or not A and B will collide again. Give a reason for your answer. *(2 marks)*

4 A small smooth ball B, of mass m, falls vertically and collides with a fixed smooth plane inclined at an angle of θ to the horizontal. The speed of B just before the collision is u. Immediately after the collision, B moves horizontally with speed v.

The diagram shows the velocities of B immediately before and after the collision.



(a) Briefly explain why the component of the velocity of B parallel to the plane is not changed by the collision. *(2 marks)*

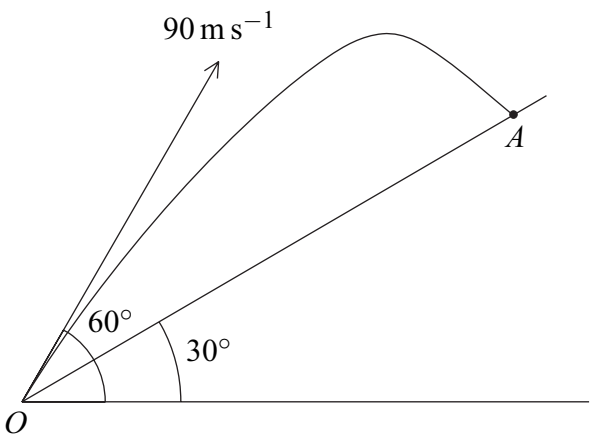
(b) Given that the coefficient of restitution between B and the plane is $\frac{3}{4}$, show that

$$3u\cos\theta = 4v\sin\theta$$ *(2 marks)*

(c) Hence show that the value of θ is approximately 40.9°. *(4 marks)*

(d) Show that the magnitude of the impulse exerted by the plane on B is approximately $1.32\,mu$. *(5 marks)*

5 A projectile is fired from a point O on the slope of a hill which is inclined at 30° to the horizontal. The projectile hits a target A. The projectile and the target are modelled as particles and the hill is modelled as a plane with OA as a line of greatest slope. The projectile is launched with a velocity of $90\,\text{m s}^{-1}$ at an inclination of 60° to the horizontal.



(a) Show that the time of the flight of the projectile is approximately 10.6 seconds. *(5 marks)*

(b) Find the distance OA. *(5 marks)*

(c) Find the maximum perpendicular distance of the projectile from the slope. *(4 marks)*

6 A golf ball is driven with speed $40\,\text{m s}^{-1}$ at an angle α to the horizontal from a point on a horizontal golf course. The ball travels in a vertical plane.

(a) Show that, during its flight, the horizontal and vertical displacements, x metres and y metres respectively, of the ball from the point of projection satisfy the equation

$$y = x\tan\alpha - \frac{gx^2}{3200}(1 + \tan^2\alpha)$$ *(5 marks)*

(b) The golf ball just clears a tree of vertical height 4 m at a horizontal distance of 100 m from its point of projection.

Find the two possible values of α. *(6 marks)*

(c) State two modelling assumptions that you have made. *(2 marks)*

END OF QUESTIONS