General Certificate of Education
June 2007
Advanced Level Examination

MATHEMATICS **MFP4**
Unit Further Pure 4

Friday 22 June 2007 9.00 am to 10.30 am

For this paper you must have:
- a 12-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 Given that $\mathbf{a} \times \mathbf{b} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$ and that $\mathbf{a} \times \mathbf{c} = -\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, determine:

(a) $\mathbf{c} \times \mathbf{a}$; *(1 mark)*

(b) $\mathbf{a} \times (\mathbf{b} + \mathbf{c})$; *(2 marks)*

(c) $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{a} \times \mathbf{c})$; *(2 marks)*

(d) $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{c})$. *(1 mark)*

2 Factorise completely the determinant $\begin{vmatrix} y & x & x+y-1 \\ x & y & 1 \\ y+1 & x+1 & 2 \end{vmatrix}$. *(6 marks)*

3 Three points, A, B and C, have position vectors

$$\mathbf{a} = \begin{bmatrix} 1 \\ 7 \\ -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{c} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

respectively.

(a) Using the scalar triple product, or otherwise, show that **a**, **b** and **c** are coplanar. *(2 marks)*

(b) (i) Calculate $(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a})$. *(3 marks)*

(ii) Hence find, to three significant figures, the area of the triangle ABC. *(3 marks)*

4 Consider the following system of equations, where k is a real constant:

$$\begin{aligned} kx &+ 2y &+ z &= 5 \\ x &+ (k+1)y &- 2z &= 3 \\ 2x &- ky &+ 3z &= -11 \end{aligned}$$

(a) Show that the system does not have a unique solution when $k^2 = 16$. *(3 marks)*

(b) In the case when $k = 4$, show that the system is inconsistent. *(4 marks)*

(c) In the case when $k = -4$:

(i) solve the system of equations; *(5 marks)*

(ii) interpret this result geometrically. *(1 mark)*

5 The line l has equation $\mathbf{r} = \begin{bmatrix} 3 \\ 26 \\ -15 \end{bmatrix} + \lambda \begin{bmatrix} 8 \\ -4 \\ 1 \end{bmatrix}$.

(a) Show that the point $P(-29, 42, -19)$ lies on l. *(1 mark)*

(b) Find:

(i) the direction cosines of l; *(2 marks)*

(ii) the acute angle between l and the z-axis. *(1 mark)*

(c) The plane Π has cartesian equation $3x - 4y + 5z = 100$.

(i) Write down a normal vector to Π. *(1 mark)*

(ii) Find the acute angle between l and this normal vector. *(4 marks)*

(d) Find the position vector of the point Q where l meets Π. *(4 marks)*

(e) Determine the shortest distance from P to Π. *(3 marks)*

Turn over for the next question

6 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & t \end{bmatrix}$$

(a) Find, in terms of t, the matrices:

(i) **AB**; *(3 marks)*

(ii) **BA**. *(2 marks)*

(b) Explain why **AB** is singular for all values of t. *(1 mark)*

(c) In the case when $t = -2$, show that the transformation with matrix **BA** is the combination of an enlargement, E, and a second transformation, F. Find the scale factor of E and give a full geometrical description of F. *(6 marks)*

7 (a) The matrix $\mathbf{M} = \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$ represents a shear.

(i) Find $\det \mathbf{M}$ and give a geometrical interpretation of this result. *(2 marks)*

(ii) Show that the characteristic equation of **M** is $\lambda^2 - 2\lambda + 1 = 0$, where λ is an eigenvalue of **M**. *(2 marks)*

(iii) Hence find an eigenvector of **M**. *(3 marks)*

(iv) Write down the equation of the line of invariant points of the shear. *(1 mark)*

(b) The matrix $\mathbf{S} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ represents a shear.

(i) Write down the characteristic equation of **S**, giving the coefficients in terms of a, b, c and d. *(2 marks)*

(ii) State the numerical value of $\det \mathbf{S}$ and hence write down an equation relating a, b, c and d. *(2 marks)*

(iii) Given that the only eigenvalue of **S** is 1, find the value of $a + d$. *(2 marks)*

END OF QUESTIONS