



General Certificate of Education

Mathematics 6360

MFP4 Further Pure 4

Mark Scheme

2008 examination – June series

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Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
✓ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP4

Q	Solution	Marks	Total	Comments
1	<p>Attempt at char eqn $\lambda^2 - 7\lambda - 144 = 0$</p> <p>Solving quadratic to find evals $\lambda = 16$ or -9</p> <p>$\lambda = 16 \Rightarrow -9x + 12y = 0 \Rightarrow y = \frac{3}{4}x$</p> <p>$\Rightarrow$ evecs $\alpha \begin{bmatrix} 4 \\ 3 \end{bmatrix}$</p> <p>$\lambda = -9 \Rightarrow 16x + 12y = 0 \Rightarrow y = -\frac{4}{3}x$</p> <p>$\Rightarrow$ evecs $\beta \begin{bmatrix} 3 \\ -4 \end{bmatrix}$</p>	<p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	6	<p>Any suitable method Ignore missing “= 0” Any method CAO</p> <p>Either λ substituted back</p> <p>CAO (for any non-zero α)</p> <p>CAO (for any non-zero β)</p>
	Total		6	
2(a)(i)	$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 2 & 1 & 2 \end{vmatrix} = \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}$	<p>M1</p> <p>A1</p>	2	<p>Genuine vector product attempt</p> <p>CAO</p>
(ii)	$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ t \\ 6 \end{bmatrix} = 4t - 20$	<p>M1</p> <p>A1</p>	2	<p>Must get a scalar answer</p> <p>ft</p>
(iii)	$(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 4 & -3 \\ -2 & t & 6 \end{vmatrix} = \begin{bmatrix} 3t + 24 \\ 0 \\ t + 8 \end{bmatrix}$	<p>M1</p> <p>A1</p>	2	<p>Either using (a)(i) or starting again</p> <p>CAO</p>
(b)	$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = 0 \Rightarrow t = 5$	M1A1	2	ft from (a)(ii)
(c)	$(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \mathbf{0} \text{ or } \mathbf{c} = \text{mult. of } (\mathbf{a} \times \mathbf{b})$ $\Rightarrow t = -8$	<p>M1</p> <p>A1</p>	2	<p>Use of any non-zero row to find some value of t</p> <p>CAO – allow unseen check</p>
	Total		10	
3(a)	$\text{Det } \mathbf{A} = k + 3 + 12 - 4 - 9 - k = 2$	<p>M1</p> <p>A1</p>	2	CAO
(b)	$\mathbf{A}^{-1} = \frac{1}{\text{Det } \mathbf{A}} (\text{adj } \mathbf{A})$ $= \frac{1}{2} \begin{bmatrix} k-9 & 3-k & 2 \\ 12-k & k-4 & -2 \\ -1 & 1 & 0 \end{bmatrix}$	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p>	5	<p>Correct use of the determinant (any value)</p> <p>Attempt at matrix of cofactors</p> <p>Use of transposition and signs</p> <p>At least 5 entries correct (even if 2nd M1 not earned)</p> <p>CAO – ft det only</p>
	Total		7	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
4(a)	Use of $\cos \theta = \frac{\text{scalar product}}{\text{product of moduli}}$ Numerator = 7 Denominator = $\sqrt{21}\sqrt{27}$ $\theta = 72.9^\circ$	M1 B1,B1 A1	 4	Must be $\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$ “sin θ =” scores M0 at this stage Allow denominator unsimplified CAO (but A0 if candidate proceeds to find its complement)
(b)(i)	$a + 4b = 7$ and $a - b = 12$ $a = 11$ and $b = -1$	B1 M1 A1	 3	At least one correctly stated Solving simultaneously CAO
(ii)	$\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \end{bmatrix} = \begin{bmatrix} 5 \\ -22 \\ 3 \end{bmatrix}$	M1 A1	 2	For any valid, complete method for finding a suitable direction vector , eg finding a 2 nd common point, eg $(2\frac{1}{2}, 0, \frac{1}{2})$ or $(1\frac{2}{3}, 3\frac{2}{3}, 0)$, and then $d\mathbf{v}$ = difference CAO
(iii)	$\mathbf{r} = \begin{bmatrix} 0 \\ 11 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 5 \\ -22 \\ 3 \end{bmatrix}$ or other equivalent line form eg $(\mathbf{r} - \mathbf{a}) \times \mathbf{d} = \mathbf{0}$	M1 A1	 2	Must be a line equation and use their (b)(ii) ft their (b)(i) point, or any other correct point on the line A0 if no $\mathbf{r} =$ or $l =$ etc
Total			11	
5(a)	$y = 0$ (or “x-axis”) and $y = x$ $y = 0$ is a line of invariant points since $\lambda = 1$	B1,B1 B1	 3	or $\mathbf{r} = a \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{r} = b \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ Allow if proven from $(x', y') = (x, y)$ or ft from their line corresponding to $\lambda = 1$
(b)(i)	$\mathbf{D} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, $\mathbf{U} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$,	B1,B1	2	ft \mathbf{U} from \mathbf{D}
(ii)	$\mathbf{U}^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ $\mathbf{M} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$ or $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$	B1 M1 A1 A1	 4	ft from \mathbf{U} (provided non-singular) Attempt ft first multiplication CAO NMS $\Rightarrow 0$

MFP4 (cont)

Q	Solution	Marks	Total	Comments
(iii)	$\mathbf{D}^n = \begin{bmatrix} 1 & 0 \\ 0 & 2^n \end{bmatrix}$ $\mathbf{M}^n = \mathbf{U} \mathbf{D}^n \mathbf{U}^{-1}$ $= \begin{bmatrix} 1 & 2^n - 1 \\ 0 & 2^n \end{bmatrix}$	B1 M1 A1	3	Noted or used Used; must actually do some multiplying
	Total		12	
6(a)	eg $(2) - (1) \Rightarrow x + 7z = -3$ $(3) - 2 \times (2) \Rightarrow x + 8z = -2$ Solving 2×2 system $x = -10, y = 19, z = 1$	M1A1 A1 M1 A1	5	Eliminating first variable
(b)(i)	$\begin{vmatrix} 1 & 1 & -3 \\ 2 & 1 & 4 \\ 5 & 2 & a \end{vmatrix} = 15 - a$ Setting = to zero and solving for a $a = 15$	B1 M1 A1	3	Determinant Must get a numerical answer ft
(ii)	$x + y - 3z = b$ $2x + y + 4z = 3$ $5x + 2y + 15z = 4$ eg $(2) - (1) \Rightarrow x + 7z = 3 - b$ $(3) - 2 \times (2) \Rightarrow x + 7z = -2$ Equating the two RHSs $b = 5$	M1A1 A1 M1 A1	5	Eliminating first variable CAO NB Eliminating x : $-y + 10z = 3 - 2b$ $-3y + 30z = 4 - 5b$ $-y + 10z = -7$ NB Eliminating z : $10x + 7y = 4b + 9$ $10x + 7y = 5b + 4$ $10x + 7y = 29$
	Total		13	
6(a)	Alternate Schemes Cramer's Rule $\Delta = \begin{vmatrix} 1 & 1 & -3 \\ 2 & 1 & 4 \\ 5 & 2 & 16 \end{vmatrix}, \Delta_x = \begin{vmatrix} 6 & 1 & -3 \\ 3 & 1 & 4 \\ 4 & 2 & 16 \end{vmatrix},$ $\Delta_y = \begin{vmatrix} 1 & 6 & -3 \\ 2 & 3 & 4 \\ 5 & 4 & 16 \end{vmatrix}, \Delta_z = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 1 & 3 \\ 5 & 2 & 4 \end{vmatrix}$ $= -1, 10, -19 \text{ and } -1 \text{ respectively}$ $x = \frac{\Delta_x}{\Delta}, y = \frac{\Delta_y}{\Delta}, z = \frac{\Delta_z}{\Delta}$ $x = -10, y = 19, z = 1$	M1 A1 M1 A1 A1	(5)	Attempt at any two Any one correct At least one attempted numerically Any 2 correct ft All 3 correct CAO

MFP4 (cont)

Q	Solution	Marks	Total	Comments
6(a)	Inverse matrix method $C^{-1} = \frac{1}{-1} \begin{bmatrix} 8 & -22 & 7 \\ -12 & 31 & -10 \\ -1 & 3 & -1 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = C^{-1} \begin{bmatrix} 6 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} -10 \\ 19 \\ 1 \end{bmatrix}$	M1 A1		M0 if no inverse matrix is given
6(all)	$\left[\begin{array}{ccc c} 1 & 1 & -3 & b \\ 2 & 1 & 4 & 3 \\ 5 & 2 & a & 4 \end{array} \right]$ $\rightarrow \left[\begin{array}{ccc c} 1 & 1 & -3 & b \\ 0 & -1 & 10 & 3-2b \\ 0 & -3 & a+15 & 4-5b \end{array} \right]$ $\rightarrow \left[\begin{array}{ccc c} 1 & 1 & -3 & b \\ 0 & 1 & -10 & 2b-3 \\ 0 & 0 & a-15 & b-5 \end{array} \right]$ <p>(b)(i) For non-unique solutions, $a = 15$</p> <p>(ii) For consistency, $4 - 5b = 3(3 - 2b) \Rightarrow b = 5$</p> <p>(a) When $a = 16, b = 6$</p> $\left[\begin{array}{ccc c} 1 & 1 & -3 & 6 \\ 0 & 1 & -10 & 9 \\ 0 & 0 & 1 & 1 \end{array} \right]$ $\Rightarrow z = 1, y = 19, x = -10$	M1 A1 A1	(5)	Any 2 correct ft All 3 correct CAO
			(4)	$R_2' = R_2 - 2R_1$ $R_3' = R_3 - 5R_1$
			(2)	$R_3' = R_3 + 3R_2$
			(2)	
			(5)	
7(a)(i)	det $\mathbf{W} = 0$ Transformed shapes have zero volume	B1 B1	2	Or equivalent statement ft volume statements
(ii)	Char eqn is $\lambda^3 - 4\lambda^2 + 4\lambda = 0$ Solving the cubic eqn $\lambda = 0, 2, (2)$	M1A3 M1 A1	 6	One A mark for each coefficient (not the λ^3)

MFP4 (cont)

Q	Solution	Marks	Total	Comments
7(a)(ii)	<p>Alternative:</p> $\text{Det } (\mathbf{W} - \lambda \mathbf{I}) = \begin{vmatrix} 3-\lambda & -1 & 1 \\ 2 & -\lambda & 2 \\ -1 & 1 & 1-\lambda \end{vmatrix}$ $= \begin{vmatrix} 2-\lambda & 0 & 2-\lambda \\ 2 & -\lambda & 2 \\ -1 & 1 & 1-\lambda \end{vmatrix}$ $= (2-\lambda) \begin{vmatrix} 1 & 0 & 1 \\ 2 & -\lambda & 2 \\ -1 & 1 & 1-\lambda \end{vmatrix}$ $= (2-\lambda) \begin{vmatrix} 1 & 0 & 0 \\ 2 & -\lambda & 0 \\ -1 & 1 & 2-\lambda \end{vmatrix}$ $= (2-\lambda)^2 \begin{vmatrix} 1 & 0 & 0 \\ 2 & -\lambda & 0 \\ -1 & 1 & 1 \end{vmatrix}$ $= (2-\lambda)^2 (-\lambda)$ <p>giving eigenvalues 0 and 2 (twice)</p>	<p>M1</p> <p>A1</p> <p>A1A1</p> <p>M1</p> <p>A1</p>		<p>Use of R/C ops. $R_1' \rightarrow R_1 + R_3$</p> <p>Factor of $(2 - \lambda)$ correctly extracted</p> <p>$C_1' \rightarrow C_3 - C_1$</p> <p>Complete factorisation attempt</p>
(b)(i)	$x - y + z = 0$	B1	1	
(ii)	$\begin{bmatrix} 3 & -1 & 1 \\ 2 & 0 & 2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3a - b + c \\ 2a + 2c \\ -a + b + c \end{bmatrix}$ $x' - y' + z' = 3a - b + c - 2a - 2c - a + b + c$ $= 0 \Rightarrow P' \text{ in } H \text{ also}$	<p>M1A1</p> <p>M1</p> <p>A1</p>	<p>4</p>	<p>Shown carefully</p>
	Total		13	
8	<p>Expanding fully: $\Delta = x^3 + y^3 + z^3 - 3xyz$</p> <p>Using row/column operations:</p> <p>eg $R_1' = R_1 + (R_2 + R_3)$</p> $\Rightarrow \Delta = (x + y + z) \begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ y & z & x \end{vmatrix}$ <p>NB Any line of argument that leads correctly from $(x + y + z) f(x, y, z)$ to $x^3 + y^3 + z^3 - 3xyz$ scores full marks</p>	<p>B1</p> <p>M1</p> <p>A1</p>	<p>3</p>	<p>With conclusion that $(x + y + z)$ is a factor of the required expression when $k = 3$</p>
	Total		3	
	TOTAL		75	