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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MFP4 Further Pure 4

Mark Scheme

2007 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP4

Q	Solution	Marks	Total	Comments
1(a)	$\mathbf{c}\times\mathbf{a} = -\,\mathbf{a}\times\mathbf{c} = \mathbf{i}+2\mathbf{j}-\mathbf{k}$	B1	1	
(b)	$\mathbf{a}\times(\mathbf{b}+\mathbf{c}) = (\mathbf{a}\times\mathbf{b})+(\mathbf{a}\times\mathbf{c})$	M1		
	$= 2\mathbf{i}-\mathbf{j}+2\mathbf{k}$	A1	2	
(c)	$(\mathbf{a}\times\mathbf{b})\,.\,(\mathbf{a}\times\mathbf{c}) = \begin{pmatrix}3\\1\\1\end{pmatrix}\bullet\begin{bmatrix}-1\\-2\\1\end{bmatrix} = -4$	M1		Must attempt to get a scalar
		A1	2	
(d)	$\mathbf{a}\,.\,(\mathbf{a}\times\mathbf{c}) = 0$ $\left(\text{since } \mathbf{a}\times\mathbf{c} \text{ perp}^{\text{r}} \text{ to } \mathbf{a}\right)$	B1	1	B0 for "0" from invalid working
			6	
2	$\Delta = \begin{vmatrix} y-x & x & x+y-1 \\ x-y & y & 1 \\ y-x & x+1 & 2 \end{vmatrix}$	M1		Attempt at first linear factor, eg $C_1{}' = C_1 - C_2$
	$= (y-x)\begin{vmatrix} 1 & x & x+y-1 \\ -1 & y & 1 \\ 1 & x+1 & 2 \end{vmatrix}$	A1		For 1st linear factor (Ignore remaining det.)
	$\Delta = (y-x)\begin{vmatrix} 0 & x+y & x+y \\ -1 & y & 1 \\ 1 & x+1 & 2 \end{vmatrix}$	M1		Attempt at second linear factor, eg $R_1{}' = R_1 + R_2$
	$= (y-x)(y+x)\begin{vmatrix} 0 & 1 & 1 \\ -1 & y & 1 \\ 1 & x+1 & 2 \end{vmatrix}$	A1		For 2nd linear factor (Ignore remaining det.)
	Full expansion	M1		
	$\Delta = (y-x)(y+x)(2-x-y)$	A1	6	Completely correct solution
	Or			
	Setting $y = x \Rightarrow C_1 = C_2 \Rightarrow \Delta = 0$	(M1)		Factor theorem
	$\Rightarrow (y-x)$ a factor of Δ	(A1)		
	Setting $y = -x \Rightarrow R_1 = R_2$	(M1)		
	So that $R_1{}' = R_1 + R_2 \Rightarrow R_1{}' = 0$			
	$\Rightarrow \Delta = 0$ and $(y+x)$ a factor of Δ	(A1)		
	Genuine attempt at 3rd factor	(M1)		
	Completely correct solution	(A1)	(6)	
	<u>Additional notes for question 2:</u> M0 for full expansion from the start with no successful factorisation progress M1 A1 M0 M1 A0 for full expansion after one factor found and remaining quadratic factor left unfactorised (or incorrectly done) 4 + M0 for two correct linear factors but final det incorrectly expanded. 5 + A0 for minor sign error, but correct otherwise			
			6	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\mathbf{a}\,.\,\mathbf{b}\times\mathbf{c} = \begin{vmatrix} 1 & 7 & -1 \\ 5 & 1 & 1 \\ 2 & -3 & 1 \end{vmatrix}$	M1		
	$= 1 + 14 + 15 + 2 + 3 - 35 = 0$	A1	2	
	Or			
	$\mathbf{b}\times\mathbf{c} = 4\mathbf{i} - 3\mathbf{j} - 17\mathbf{k}$ and $\begin{bmatrix} 4 \\ -3 \\ -17 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ -1 \end{bmatrix} = 0$	(M1) (A1)	(2)	Or equivalent
	Or			
	$\mathbf{b} = \mathbf{a} + 2\mathbf{c} \Rightarrow$ co-planarity	(M1) (A1)	(2)	
(b)(i)	$\mathbf{b} - \mathbf{a} = 4\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$ $\quad \mathbf{c} - \mathbf{a} = \mathbf{i} - 10\mathbf{j} + 2\mathbf{k}$	B1		Either correct
	$(\mathbf{b} - \mathbf{a})\times(\mathbf{c} - \mathbf{a}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -6 & 2 \\ 1 & -10 & 2 \end{vmatrix}$	M1		Genuine attempt using their two vectors
	$= 8\mathbf{i} - 6\mathbf{j} - 34\mathbf{k}$	A1	3	CSO
(ii)	Area $\Delta ABC = \frac{1}{2}$ | this vector |	M1		Must be "Hence" method
	$= \frac{1}{2}\times 2\sqrt{4^2 + 3^2 + 17^2}$	M1		Correct modulus attempt
	$= \sqrt{314}$ or $17.7(2)$	A1$\sqrt{}$	3	ft (b)(i) only
			8	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\Delta = \begin{vmatrix} k & 2 & 1 \\ 1 & k+1 & -2 \\ 2 & -k & 3 \end{vmatrix}$			
	$= 3k^2 + 3k - k - 8 - 2(k+1) - 2k^2 - 6$	M1		Genuine attempt at Δ
	$= k^2 - 16$	A1		
	When $k^2 = 16$ $\Delta = 0$ $\Rightarrow$ no unique soln.	E1	3	Explained
	Or Subst^g. **Both** $k = 4$ and $k = -4$ and attempt at det.	(M1)		
	Each case correctly shown	(A1)		
		(A1)	3	
(b)	$4x + 2y + z = 5$ $k = 4 \Rightarrow$ $x + 5y - 2z = 3$ $2x - 4y + 3z = -11$	B1		
	Elim^g. z from **(1) & (2)** $\Rightarrow$ $9(x + y) = 13$	M1		Eliminating one variable
	(1) & (3) $\Rightarrow$ $10(x + y) = 26$	A1		Twice, correctly
	Or **(2) & (3)** $\Rightarrow$ $7(x + y) = -13$			
	Explaining inconsistency,eg from $\frac{13}{9} \neq \frac{26}{10}$	E1	4	
	Alternatively (mark as above) Elim^g. x from **(1) & (2)** $\Rightarrow$ $9(2y - z) = 7$ **(2) & (3)** $\Rightarrow$ $7(2y - z) = 17$ **(1) & (3)** $\Rightarrow$ $5(2y - z) = 27$ **Or** Elim^g. y from **(1) & (2)** $\Rightarrow$ $9(2x + z) = 19$ **(2) & (3)** $\Rightarrow$ $7(2x + z) = -43$ **(1) & (3)** $\Rightarrow$ $5(2x + z) = -1$			
(c)(i)	$-4x + 2y + z = 5$ $k = -4 \Rightarrow$ $x - 3y - 2z = 3$ $2x + 4y + 3z = -11$	B1		
	Eliminating one variable	M1		Any pair of equations
	$-7x + y = 13$ **Or** $10y + 7z = -17$ **Or** $10x + z = -21$	A1		Correct
	Parametrisation	M1		Or equivalent
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 13 \\ -21 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \\ -10 \end{pmatrix}$	A1	5	Any correct answer in any form
	Correct alternate answer forms: x, $y = 13 + 7x$, $z = -21 - 10x$ y, $x = (y - 13) / 7$, $z = (-21 - 10y) / 7$ z, $y = (-17 - 7z) / 10$, $x = (-21 - z) / 10$ Do not accept a mixed parametrisation			Or equivalents
(ii)	The line of intersection of 3 planes	B1	1	Or "Sheaf" of planes
			13	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\lambda = -4$ gives $P(-29, 42, -19)$ on l	B1	1	Correct value of λ
(b)(i)	$\sqrt{8^2+4^2+1^2} = 9$	B1		Can be awarded retrospectively in (b)(ii) if (b)(i) not done
	dir. cos.s are $\frac{8}{9}, -\frac{4}{9}, \frac{1}{9}$	B1√	2	ft denom^r.
(ii)	$\cos^{-1}\frac{1}{9}$ or 83.6° (or 84°) or 1.46 rads.	B1√	1	ft from 3rd d.c. or by any other method (e.g. scalar product) N.B. Mark lost if 6.4° is then offered as the answer
(c)(i)	$\mathbf{n} = 3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$	B1	1	
(ii)	Use of $\cos\theta = \frac{\text{scalar product}}{\text{product of moduli}}$	M1		Must be direction vector of l and their $\mathbf{n}$
	Nr. = 45 Dr. = $\sqrt{50}\,.9$	A1 A1		ft the "9" if necessary from (b) (i)
	$\theta = 45°$	A1	4	CAO
(d)	Subst^g. $\begin{pmatrix} 3+8\lambda \\ 26-4\lambda \\ \lambda-15 \end{pmatrix}$ in $3x - 4y + 5z = 100$	M1		$3(3 + 8\lambda) - 4(26 - 4\lambda) + 5(\lambda - 15) = 100$
	Solving a linear eqn. in λ	dM1		
	$\lambda = 6$	A1		CAO
	$\Rightarrow Q = (51, 2, -9)$	B1√	4	ft their λ in l
(e)	$PQ = \sqrt{80^2+40^2+10^2} = 90$	B1		ft
	Sh. Dist.= 90 sin 45° = $45\sqrt{2}$ or 63.6(4…)	M1 A1√	3	ft
	Or $\mathbf{p} + m\,\mathbf{n}$ subst^d. into $\Pi \Rightarrow m = 9$	(M1)		
	$\Rightarrow R = (-2, 6, 26)$	(A1)		R = foot of perp^r. from P to Π
	$PR = \sqrt{27^2+36^2+45^2} = 45\sqrt{2}$	(B1√)	(3)	ft
			16	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	**AB** = a 3×3 matrix	M1		
	$= \begin{pmatrix} 3 & 2 & t+1 \\ 1 & 2 & t-1 \\ 3 & 2 & t+1 \end{pmatrix}$	A1 A1	3	At least 5 elements correct, incl. at least one from C_3 All elements correct
(ii)	**BA** = a 2×2 matrix	M1		
	$= \begin{pmatrix} 2 & 2 \\ t & t+4 \end{pmatrix}$	A1	2	
(b)	$R_1 = R_3 \left(\Rightarrow \det \mathbf{AB} = 0 \right)$	B1	1	Or expanding and **showing** $\det = 0$
(c)	**BA** $= \begin{pmatrix} 2 & 2 \\ -2 & 2 \end{pmatrix} = 2\sqrt{2} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$	M1 A1		
	E: enlargement s.f. $2\sqrt{2}$	B1		
	F: Rotation	M1		NB: Rotation bit may be sorted completely separately in which case marks are split 3 + 3
	clockwise (about O) thro' 45°	A1 A1	6	Or -45°, 315°
			12	
7(a)(i)	det **M** = 1 $\Rightarrow$ **area** invariant	B1B1	2	
(ii)	$\lambda^2 - (\text{trace } \mathbf{M})\lambda + (\det \mathbf{M}) = 0$	M1 A1	2	Answer given; condone lack of "= 0"
(iii)	$\lambda = 1$ subst^{d}. back $\Rightarrow -2x + 2y = 0$	M1 A1		
	and evec. is $\alpha \begin{pmatrix} 1 \\ 1 \end{pmatrix}$	A1	3	Any non-zero multiple will do
(iv)	$y = x$ (since $\lambda = 1$) or vector eqn.	B1	1	CAO unless following obviously incorrect working
(b)(i)	$\lambda^2 - (a+d)\lambda + (ad - bc) = 0$	B1 B1	2	Including "= 0" here to be an eqn.
(ii)	det **S** = 1	B1		
	$\Rightarrow ad - bc = 1$	B1√	2	ft 2nd B1 from numerical det **S**
(iii)	λ=1 twice gives Char. Eqn. $\lambda^2 - 2\lambda + 1 = 0$	M1		
	$\Rightarrow a + d = 2$	A1	2	CSO
	Or Subst^{g}. $\lambda = 1$ in Char. Eqn.			
	$\Rightarrow 1 - (a+d) + (ad - bc) = 0$	(M1)		
	and $ad - bc = 1 \Rightarrow a + d = 2$	(A1)	(2)	CSO
	Total		**14**	
	TOTAL		**75**	