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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MFP4 Further Pure 4

Mark Scheme

2007 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP4

Q	Solution	Marks	Total	Comments
1	$\left(\begin{array}{ccc\|c} 1 & 2 & -1 & 0 \\ 3 & -1 & 4 & 7 \\ 8 & 1 & 7 & 30 \end{array}\right) \to \left(\begin{array}{ccc\|c} 1 & 2 & -1 & 0 \\ 0 & -1 & 7 & 7 \\ 0 & -15 & 15 & 30 \end{array}\right)$	M1 A1		$R_2' = R_2 - 3R_1$ $R_3' = R_3 - 8R_1$ Penalise numerical errors once only, at this stage
	$\to \left(\begin{array}{ccc\|c} 1 & 2 & -1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 2 \end{array}\right)$	A1 B1F	4	Inconsistency noted/explained ft provided working is clear
	Or $\Delta = -7 - 3 + 64 - 8 - 4 - 42 = 0$ and Δ_x or Δ_y or $\Delta_z = 0$ shown also Explaining this $\Rightarrow$ inconsistency	(M1) (A1) (A1) (B1)	(4)	So showing $\Delta = 0$ and thinking this is it scores M1A1A0B0
	Or Solving **(1)** & **(2)**, say, to get $x = \lambda,\ y = 1 - \lambda,\ z = 2 - \lambda$ Subst^{g}. in **(3)** $\Rightarrow$ $15 = 30$	(M1) (A1) (A1) (B1)	(4)	Checking to show inconsistency
	Total		**4**	
2(a)	$\Delta = \begin{vmatrix} a-b & b & c \\ b-a & c+a & a+b \\ c(b-a) & ca & ab \end{vmatrix}$	M1		$C_1' = C_1 - C_2$
	$= (a-b)\begin{vmatrix} 1 & b & c \\ -1 & c+a & a+b \\ -c & ca & ab \end{vmatrix}$	A1	2	
	Or Setting $b = a \Rightarrow C_1 = C_2 \Rightarrow \Delta = 0$ $\Rightarrow (a-b)$ a factor of Δ	(M1) (A1)	(2)	Factor theorem
	Or $\Delta = (a-b)(c^3 + a^2b + ab^2 - abc - b^2c - a^2c)$	(M1) (A1)	(2)	Must be completely correct

MFP4 (cont)

Q	Solution	Marks	Total	Comments
2(b)	$= (a-b)\begin{vmatrix} 1 & b-c & c \\ -1 & c-b & a+b \\ -c & a(c-b) & ab \end{vmatrix}$	M1		$C_2' = C_2 - C_3$
	$= (a-b)(b-c)\begin{vmatrix} 1 & 1 & c \\ -1 & -1 & a+b \\ -c & -a & ab \end{vmatrix}$	A1		2nd linear factor extracted
	e.g. $\Delta = (a-b)(b-c)\begin{vmatrix} 0 & 0 & a+b+c \\ -1 & -1 & a+b \\ -c & -a & ab \end{vmatrix}$	M1		Genuine attempt at both remaining linear factors: e.g. $R_1' = R_1 + R_2$
	and then expanding final det.	A1		3rd factor
	$\Delta = -(a+b+c)(a-b)(b-c)(c-a)$	A1	5	All correct
	Or By cyclic symmetry, $(b-c)$ and $(c-a)$ are also factors	(M1) (A1) (A1)		
	Final linear factor & checking sign of a coefficient.	(M1) (A1)	(5)	
	Or Expanding the determinant fully	(M1)		
	$\Delta =$	(A1)		
	Multiplying out $(a-b)(b-c)(c-a)(a+b+c)$	(M1)		No fudging, or jumping straight to the answer allowed
	$=$	(A1)		
	Fully correct working to show the two things are identically equal & checking for sign	(A1)	(5)	
	Total		**7**	
3(a)(i)	$\mathbf{p} \times \mathbf{q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 4 \\ -3 & 4 & 20 \end{vmatrix} = \begin{bmatrix} 4 \\ -32 \\ 7 \end{bmatrix}$	M1 A1	2	
(ii)	$A = \frac{1}{2} \lvert \mathbf{p} \times \mathbf{q} \rvert$	M1		
	$= \frac{1}{2}\sqrt{4^2 + 32^2 + 7^2}$	B1		For attempt at $\lvert \mathbf{p} \times \mathbf{q} \rvert$
	$= \frac{33}{2}$	A1F	3	ft
(b)	$\mathbf{p} \times \mathbf{q} \cdot \mathbf{r} = \begin{bmatrix} 4 \\ -32 \\ 7 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 2 \\ 4 \end{bmatrix}$ *or* $\begin{vmatrix} 1 & 1 & 4 \\ -3 & 4 & 20 \\ 9 & 2 & 4 \end{vmatrix}$	M1		
	$= 36 - 64 + 28 = 0$	A1		Give when "= 0" reached
	($\Rightarrow$ Lin Dep) O, P, Q, R **Or** **p**, **q**, **r** co-planar	B1	3	
	Total		**8**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
4(a)	A is a Rotation thro' 90°	M1 A1		
	about Ox	A1		
	B is a Reflection in $y = 0$ (i.e. x–z plane)	M1 A1	5	
(b)(i)	$\mathbf{M}_C = \mathbf{M}_B\,\mathbf{M}_A = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$	M1 A1	2	
(ii)	C is a Reflection in $y = z$	M1	2	Give M1 for any series of reflections
		A1		
	N.B. In (i):			
	$\mathbf{M}_A\,\mathbf{M}_B = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & -1 & 0\end{bmatrix}$ scores M0			
	but ft "Reflection in $y = -z$" in (ii)			
			9	
5(a)	Use of $\cos\theta = \dfrac{\text{scalar product}}{\text{product of moduli}}$	M1		Must be $(3\mathbf{i} - 4\mathbf{j} + \mathbf{k})$ and $(2\mathbf{i} + 12\mathbf{j} - \mathbf{k})$
	Numerator $= \pm 43$	B1		
	Denominator $= \sqrt{26}\,.\,\sqrt{149}$	B1		Dr. $= 5.099... \times 3.742... = 0.6908...$
	$\theta = 46.3^\circ$	A1	4	
(b)	$3x - 4y + z = 2$ and $2x + 12y - z = 38$	B1 B1	2	
(c)(i)	$(3\mathbf{i} - 4\mathbf{j} + \mathbf{k}) \times (2\mathbf{i} + 12\mathbf{j} - \mathbf{k})$	M1		
	$= -8\mathbf{i} + 5\mathbf{j} + 44\mathbf{k}$	A1		
	p.v. of any point on line	M1		
	e.g. (0, 5, 22), (8, 0, – 22), $(4, 2\frac{1}{2}, 0)$	A1		
	$\dfrac{x - x_c}{-8} = \dfrac{y - y_c}{5} = \dfrac{z - z_c}{44}$	B1F	5	ft
	Or Adding $\Rightarrow 5x + 8y = 40$ (e.g.)	(M1)		Eliminating one variable
	$\dfrac{x-8}{-8} = \dfrac{y}{5} = \lambda$ Or $\dfrac{x}{-8} = \dfrac{y-5}{5} = \mu$	(dM1)		Parametrisation attempted
		(A1)		
	$x = 8 - 8\lambda$, $x = -8\mu$			
	$y = 5\lambda$ $y = 5 + 5\mu$	(M1)		Subst^g. to find third variable
	$\Rightarrow z = 44\lambda - 22$ $\Rightarrow z = 44\lambda + 22$			
	$\dfrac{x - x_c}{-8} = \dfrac{y - y_c}{5} = \dfrac{z - z_c}{44}$	(A1)	(5)	
(ii)	$\sqrt{8^2 + 5^2 + 44^2} = 45$	B1F		ft
	d.c.s are $\dfrac{-8}{45}$, $\dfrac{1}{9}$ and $\dfrac{44}{45}$	B1F	2	ft
	Total		**13**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
6(a)	Char. Eqn. is $\lambda^2 - 5\lambda - 6 = 0$	B1		
	Solving $\Rightarrow \lambda = -1$ or 6	M1 A1		
	Subst^g. either λ back	M1		
	$\lambda = -1 \Rightarrow x + y = 0 \Rightarrow$ evecs. $\alpha\begin{bmatrix}1\\-1\end{bmatrix}$	A1		Any non-zero α
	$\lambda = 6 \Rightarrow 5x - 2y = 0 \Rightarrow$ evecs. $\beta\begin{bmatrix}2\\5\end{bmatrix}$	A1	6	Any non-zero β
(b)(i)	$\mathbf{D} = \begin{bmatrix}-1 & 0\\0 & 6\end{bmatrix}$ $\quad \mathbf{U} = \begin{bmatrix}1 & 2\\-1 & 5\end{bmatrix}$	B1F B1F	2	ft evals. ft evecs. (must correspond to their evals.)
(ii)	$\mathbf{U}^{-1} = \frac{1}{7}\begin{bmatrix}5 & -2\\1 & 1\end{bmatrix}$	B1F	1	ft their **U** (provided non-singular)
(iii)	$\mathbf{X}^5 = \mathbf{U}\,\mathbf{D}^5\,\mathbf{U}^{-1}$	M1		
	$= \frac{1}{7}\begin{bmatrix}1 & 2\\-1 & 5\end{bmatrix}\begin{bmatrix}-1 & 0\\0 & 6^5\end{bmatrix}\begin{bmatrix}5 & -2\\1 & 1\end{bmatrix}$	B1F		Use of correct $\mathbf{D}^5$ (ft) N.B. $6^5 = 7776$
	$= \begin{bmatrix}2221 & 2222\\5555 & 5554\end{bmatrix}$	A1	3	
			12	
7(a)	Setting $x' = x$ and $y' = y$ $x = -x + 2y$ and $y = -2x + 3y$	M1		Or via evals/evecs
	gives $y = x$	A1	2	
(b)	$\begin{bmatrix}-1 & 2\\-2 & 3\end{bmatrix}\begin{bmatrix}x\\x+c\end{bmatrix} = \begin{bmatrix}x+2c\\x+3c\end{bmatrix}$	M1A1		
	And $y' = x' + c$ also	B1	3	Explanation
(c)	det $\mathbf{M} = 1 \Rightarrow$ Areas of shapes invariant	B1 B1	2	
(d)	$\begin{bmatrix}-1 & 2\\-2 & 3\end{bmatrix}\begin{bmatrix}a\\-a\end{bmatrix} = \begin{bmatrix}-3a\\-5a\end{bmatrix}$	M1		
	$\Rightarrow$ Image of $y = -x$ under S is $y = \frac{5}{3}x$	A1		
	Angle is $135° - \tan^{-1}\frac{5}{3} = 76°$	B1F	3	ft
	N.B. Final angle can be gained via scalar product: $\cos\theta = \left\lvert\frac{(\mathbf{i}-\mathbf{j})\cdot(-3\mathbf{i}-5\mathbf{j})}{\sqrt{2}\sqrt{34}}\right\rvert$ $\Rightarrow \theta = \cos^{-1}(1/\sqrt{17}) = 76°$			
	Total		**10**	

MFP4 (Cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	det $\mathbf{P} = 4a + 6 + 4 + a = 5a + 10$	M1 A1	2	
(ii)	When $a = 3$, det $\mathbf{P} = 25$	B1F	1	ft
(iii)	Setting their det $\mathbf{P} = 0 \Rightarrow a = -2$	M1 A1F	2	ft
(b)(i)	$\mathbf{P}^{-1} = \frac{1}{25}\mathbf{Q}$	B1	1	
(ii)	$(\mathbf{PQ})^{-1} = (25\,\mathbf{I})^{-1} = \frac{1}{25}\mathbf{I}$	M1 A1	2	
	Or $(\mathbf{PQ})^{-1} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$	(M1)		Ignore $(\mathbf{PQ})^{-1} = \mathbf{P}^{-1}\mathbf{Q}^{-1}$ if they can make it work
	$= \mathbf{Q}^{-1}.\frac{1}{25}\mathbf{Q} = \frac{1}{25}\mathbf{I}$	(A1)	(2)	
(iii)	det $\mathbf{PQ}$ = det $(25\,\mathbf{I}) = 25^3$ or 15625	M1 A1		
	det $\mathbf{PQ}$ = det $\mathbf{P}$. det $\mathbf{Q}$	M1		Used
	$\Rightarrow 25^3 = 25$ det $\mathbf{Q}$			
	$\Rightarrow$ det $\mathbf{Q} = 25^2$ or 625	A1	4	
	Total		**12**	
	TOTAL		**75**	