

General Certificate of Education
June 2006
Advanced Level Examination



MATHEMATICS
Unit Further Pure 3

MFP3

Monday 19 June 2006 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
 - the **blue** AQA booklet of formulae and statistical tables
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 It is given that y satisfies the differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = 8x - 10 - 10\cos 2x$$

- (a) Show that $y = 2x + \sin 2x$ is a particular integral of the given differential equation. *(3 marks)*
- (b) Find the general solution of the differential equation. *(4 marks)*
- (c) Hence express y in terms of x , given that $y = 2$ and $\frac{dy}{dx} = 0$ when $x = 0$. *(4 marks)*

2 The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where

$$f(x, y) = \frac{x^2 + y^2}{xy}$$

and

$$y(1) = 2$$

- (a) Use the Euler formula

$$y_{r+1} = y_r + hf(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(1.1)$. *(3 marks)*

- (b) Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = hf(x_r, y_r)$ and $k_2 = hf(x_r + h, y_r + k_1)$ and $h = 0.1$, to obtain an approximation to $y(1.1)$, giving your answer to four decimal places. *(6 marks)*

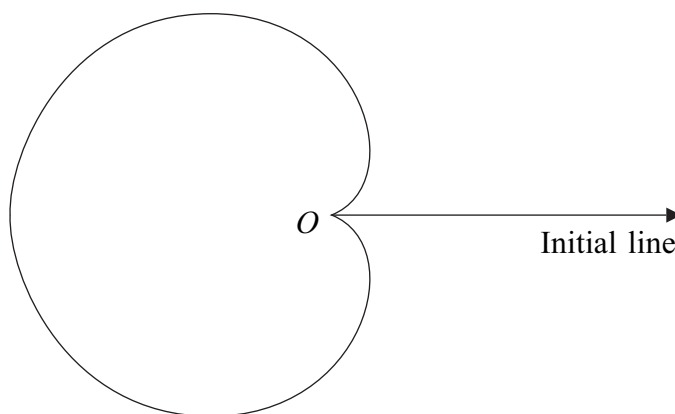
- 3 (a) Show that $\sin x$ is an integrating factor for the differential equation

$$\frac{dy}{dx} + (\cot x)y = 2 \cos x \quad (3 \text{ marks})$$

- (b) Solve this differential equation, given that $y = 2$ when $x = \frac{\pi}{2}$. (6 marks)

- 4 The diagram shows the curve C with polar equation

$$r = 6(1 - \cos \theta), \quad 0 \leq \theta < 2\pi$$



- (a) Find the area of the region bounded by the curve C . (6 marks)

- (b) The circle with cartesian equation $x^2 + y^2 = 9$ intersects the curve C at the points A and B .

- (i) Find the polar coordinates of A and B . (4 marks)

- (ii) Find, in surd form, the length of AB . (2 marks)

- 5 (a) Show that $\lim_{a \rightarrow \infty} \left(\frac{3a+2}{2a+3} \right) = \frac{3}{2}$. (2 marks)

- (b) Evaluate $\int_1^{\infty} \left(\frac{3}{3x+2} - \frac{2}{2x+3} \right) dx$, giving your answer in the form $\ln k$, where k is a rational number. (5 marks)

- 6 (a) Show that the substitution

$$u = \frac{dy}{dx} + 2y$$

transforms the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-2x}$$

into

$$\frac{du}{dx} + 2u = e^{-2x} \quad (4 \text{ marks})$$

- (b) By using an integrating factor, or otherwise, find the general solution of

$$\frac{du}{dx} + 2u = e^{-2x}$$

giving your answer in the form $u = f(x)$. (5 marks)

- (c) Hence find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-2x}$$

giving your answer in the form $y = g(x)$. (5 marks)

- 7 (a) (i) Write down the first three terms of the binomial expansion of $(1 + y)^{-1}$, in ascending powers of y . *(1 mark)*

- (ii) By using the expansion

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

and your answer to part (a)(i), or otherwise, show that the first three non-zero terms in the expansion, in ascending powers of x , of $\sec x$ are

$$1 + \frac{x^2}{2} + \frac{5x^4}{24} \quad (5 \text{ marks})$$

- (b) By using Maclaurin's theorem, or otherwise, show that the first two non-zero terms in the expansion, in ascending powers of x , of $\tan x$ are

$$x + \frac{x^3}{3} \quad (3 \text{ marks})$$

- (c) Hence find $\lim_{x \rightarrow 0} \left(\frac{x \tan 2x}{\sec x - 1} \right)$. *(4 marks)*

END OF QUESTIONS

There are no questions printed on this page

There are no questions printed on this page

There are no questions printed on this page