

General Certificate of Education
January 2006
Advanced Level Examination



MATHEMATICS
Unit Further Pure 3

MFP3

Friday 27 January 2006 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Find the roots of the equation $m^2 + 2m + 2 = 0$ in the form $a + ib$. (2 marks)

(b) (i) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 4x \quad (6 \text{ marks})$$

(ii) Hence express y in terms of x , given that $y = 1$ and $\frac{dy}{dx} = 2$ when $x = 0$. (4 marks)

2 (a) Find $\int_0^a xe^{-2x} dx$, where $a > 0$. (5 marks)

(b) Write down the value of $\lim_{a \rightarrow \infty} a^k e^{-2a}$, where k is a positive constant. (1 mark)

(c) Hence find $\int_0^{\infty} xe^{-2x} dx$. (2 marks)

3 (a) Show that $y = x^3 - x$ is a particular integral of the differential equation

$$\frac{dy}{dx} + \frac{2xy}{x^2 - 1} = 5x^2 - 1 \quad (3 \text{ marks})$$

(b) By differentiating $(x^2 - 1)y = c$ implicitly, where y is a function of x and c is a constant, show that $y = \frac{c}{x^2 - 1}$ is a solution of the differential equation

$$\frac{dy}{dx} + \frac{2xy}{x^2 - 1} = 0 \quad (3 \text{ marks})$$

(c) Hence find the general solution of

$$\frac{dy}{dx} + \frac{2xy}{x^2 - 1} = 5x^2 - 1 \quad (2 \text{ marks})$$

- 4 (a) Use the series expansion

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots$$

to write down the first four terms in the expansion, in ascending powers of x , of $\ln(1-x)$. (1 mark)

- (b) The function f is defined by

$$f(x) = e^{\sin x}$$

Use Maclaurin's theorem to show that when $f(x)$ is expanded in ascending powers of x :

- (i) the first three terms are

$$1 + x + \frac{1}{2}x^2 \quad (6 \text{ marks})$$

- (ii) the coefficient of x^3 is zero. (3 marks)

- (c) Find

$$\lim_{x \rightarrow 0} \frac{e^{\sin x} - 1 + \ln(1-x)}{x^2 \sin x} \quad (4 \text{ marks})$$

Turn over for the next question

- 5 (a) The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $f(x, y) = x \ln x + \frac{y}{x}$

and $y(1) = 1$

- (i) Use the Euler formula

$$y_{r+1} = y_r + h f(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(1.1)$. *(3 marks)*

- (ii) Use the formula

$$y_{r+1} = y_{r-1} + 2h f(x_r, y_r)$$

with your answer to part (a)(i) to obtain an approximation to $y(1.2)$, giving your answer to three decimal places. *(4 marks)*

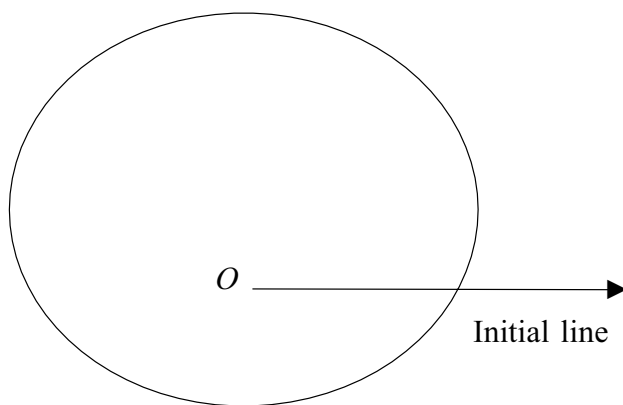
- (b) (i) Show that $\frac{1}{x}$ is an integrating factor for the first-order differential equation

$$\frac{dy}{dx} - \frac{1}{x}y = x \ln x \quad (3 \text{ marks})$$

- (ii) Solve this differential equation, given that $y = 1$ when $x = 1$. *(6 marks)*

- (iii) Calculate the value of y when $x = 1.2$, giving your answer to three decimal places. *(1 mark)*

- 6 (a) A circle C_1 has cartesian equation $x^2 + (y - 6)^2 = 36$. Show that the polar equation of C_1 is $r = 12 \sin \theta$. (4 marks)
- (b) A curve C_2 with polar equation $r = 2 \sin \theta + 5$, $0 \leq \theta \leq 2\pi$ is shown in the diagram.



- Calculate the area bounded by C_2 . (6 marks)
- (c) The circle C_1 intersects the curve C_2 at the points P and Q . Find, in surd form, the area of the quadrilateral $OPMQ$, where M is the centre of the circle and O is the pole. (6 marks)

END OF QUESTIONS

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