ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MFP3 Further Pure 3

Mark Scheme

2007 examination - January series

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP3

Q	Solution	Marks	Total	Comments
1(a)	$y\ (1.05) = 0.6 + 0.05 \times [\ln(1 + 1 + 0.6)]$	M1A1		
	$= 0.6477(7557..) = 0.6478$ to 4dp	A1	3	Condone >4 dp
(b)	$k_1 = 0.05 \times \ln(1 + 1 + 0.6) = 0.0477(75\ldots)$	M1		PI
		A1F		ft candidate's evaluation in (a)
	$k_2 = 0.05 \times \mathrm{f}\ (1.05,\ 0.6477\ldots)$			
	$\ldots = 0.05 \times \ln(1 + 1.05^2 + 0.6477\ldots)]$	M1		
	$\ldots = 0.0505(85\ldots)$	A1F		PI
	$y(1.05) = y(1) + \frac{1}{2}[k_1 + k_2]$	m1		Dep on previous two Ms and numerical values for k's
	$= 0.6 + 0.5 \times 0.09836\ldots$			
	$= 0.6492$ to 4dp	A1F	6	Must be 4 dp… ft one slip
	Total		**9**	
2	$r - r\sin\theta = 4$	M1		
	$r - y = 4$	B1		$r\sin\theta = y$ stated or used
	$r = y + 4$	A1		
	$x^2 + y^2 = (y + 4)^2$	M1		$r^2 = x^2 + y^2$ used
	$x^2 + y^2 = y^2 + 8y + 16$	A1F		ft one slip
	$y = \dfrac{x^2 - 16}{8}$	A1	6	
	Total		**6**	
3(a)	IF is $\exp\left(\int \frac{2}{x}\mathrm{d}x\right)$	M1		And with integration attempted
	$= \mathrm{e}^{2\ln x}$	A1		
	$= x^2$	A1	3	CSO **AG** be convinced
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\left[yx^2\right] = 3x^2(x^3 + 1)^{\frac{1}{2}}$	M1A1		PI
	$\Rightarrow yx^2 = \frac{2}{3}(x^3 + 1)^{\frac{3}{2}} + A$	m1		$k(x^3 + 1)^{\frac{3}{2}}$
		A1		Condone missing 'A'
	$\Rightarrow 4 = \frac{2}{3}(9)^{\frac{3}{2}} + A$	m1		Use of boundary conditions to find constant
	$\Rightarrow A = -14$			
	$\Rightarrow y = x^{-2}\left\{\frac{2}{3}(x^3 + 1)^{\frac{3}{2}} - 14\right\}$	A1	6	Any correct form
	Total		**9**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	Integrand is not defined at $x = 0$	E1	1	OE
(b)	$\int x^{-\frac{1}{2}} \ln x \, \mathrm{d}x = 2x^{\frac{1}{2}} \ln x - \int 2x^{\frac{1}{2}} \left(\frac{1}{x}\right) \mathrm{d}x$	M1		$\ldots = kx^{\frac{1}{2}} \ln x \pm \int \mathrm{f}(x)$, with $\mathrm{f}(x)$ not involving the 'original' $\ln x$
		A1		
	$\ldots\ldots = 2x^{\frac{1}{2}} \ln x - 4x^{\frac{1}{2}} (+c)$	A1	3	Condone absence of '$+ c$'
(c)	$\int_0^{\mathrm{e}} \frac{\ln x}{\sqrt{x}} \, \mathrm{d}x = \lim_{a \to 0} \int_a^{\mathrm{e}} \frac{\ln x}{\sqrt{x}} \, \mathrm{d}x$	M1		
	$= -2\mathrm{e}^{\frac{1}{2}} - \lim_{a \to 0} \left[2a^{\frac{1}{2}} \ln a - 4a^{\frac{1}{2}} \right]$	M1		$\mathrm{F}(b) - \mathrm{F}(a)$
	But $\lim_{a \to 0} a^{\frac{1}{2}} \ln a = 0$	B1		Accept a general form e.g. $\lim_{x \to 0} x^k \ln x = 0$
	So $\int_0^{\mathrm{e}} \frac{\ln x}{\sqrt{x}} \, \mathrm{d}x$ exists and $= -2\mathrm{e}^{\frac{1}{2}}$	A1	4	
	Total		**8**	
5	Auxl. eqn $m^2 - 4m + 3 = 0$	M1		PI
	$m = 3$ and 1	A1		PI
	CF is $A\mathrm{e}^{3x} + B\mathrm{e}^{x}$	A1F		
	PI Try $y = a + b\sin x + c\cos x$	M1		Condone 'a' missing here
	$y'(x) = b\cos x - c\sin x$	A1		
	$y''(x) = -b\sin x - c\cos x$	A1F		ft can be consistent sign error(s)
	Substitute into DE gives	M1		
	$a = 2$	B1		
	$4c + 2b = 5$ and $2c - 4b = 0$	A1		
	$b = 0.5$,	A1F		ft a slip
	$c = 1$	A1F		ft a slip
	GS: $y = A\mathrm{e}^{3x} + B\mathrm{e}^{x} + 2 + 0.5\sin x + \cos x$	B1F	12	$y =$ candidate's CF and candidate's PI (must have exactly two arbitrary constants)
	Total		**12**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$f'(x)=\frac{1}{2}(1+2x)^{-\frac{1}{2}}(2)=(1+2x)^{-\frac{1}{2}}$	M1A1		
	$f''(x)=-(1+2x)^{-\frac{3}{2}}$	A1F		ft a slip
	$f'''(x)=3(1+2x)^{-\frac{5}{2}}$	A1	4	
(ii)	$f(x)=(1+2x)^{\frac{1}{2}} \Rightarrow f(0)=1;$	B1		
	$f'(0)=1;\ f''(0)=-1;\ f'''(0)=3$	M1		All three attempted
		A1F		ft on $k(1+2x)^{m}$
	$f(x)=f(0)+xf'(0)+\frac{x^2}{2}f''(0)+\frac{x^3}{6}f'''(0)$			
	$\ldots\approx 1+x-\frac{x^2}{2}+\frac{x^3}{2}$	A1	4	CSO **AG**
(b)	$e^x(1+2x)^{\frac{1}{2}}\approx$			
	$\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\right)\left(1+x-\frac{x^2}{2}+\frac{x^3}{2}\right)$	M1		Attempt to expand needed
	$\approx 1+x\,(1+1)+x^2(-0.5+1+0.5)$	A1		
	$+\ x^3\left(\frac{1}{2}-\frac{1}{2}+\frac{1}{2}+\frac{1}{6}\right)$			
	$\approx 1+2x+x^2+\frac{2}{3}x^3$	A1	3	CSO
(c)	$e^{2x}=1+2x+\frac{(2x)^2}{2}+\frac{(2x)^3}{6}+\ldots$	B1	1	
	$=1+2x+2x^2+\frac{4}{3}x^3+\ldots$			
(d)	$1-\cos x=\frac{1}{2}x^2+\{o(x^4)\}$	B1		
	$\frac{e^x(1+2x)^{\frac{1}{2}}-e^{2x}}{1-\cos x}=$			
	$\frac{1+2x+x^2+\frac{2}{3}x^3-\left[1+2x+2x^2+\frac{4}{3}x^3\right]}{\frac{1}{2}x^2+\{o(x^4)\}}$	M1		Series used
	$\lim_{x\to 0}\ldots\ldots=\lim_{x\to 0}\frac{-x^2+\{o(x^3)\}}{\frac{1}{2}x^2+\{o(x^4)\}}=$	A1F		
	$\lim_{x\to 0}\frac{-1+o(x)}{\frac{1}{2}+o(x^2)}=-2$	A1F	4	ft a slip but must see the intermediate stage
	Total		**16**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	Area = $\frac{1}{2}\int(6+4\cos\theta)^2\,d\theta$	M1		use of $\frac{1}{2}\int r^2\,d\theta$
	$=\frac{1}{2}\left(\int_{-\pi}^{\pi}36+48\cos\theta+16\cos^2\theta\right)d\theta$	B1 B1		for correct expansion of $[6+4\cos\theta)]^2$ for limits
	$=\left(\int_{-\pi}^{\pi}18+24\cos\theta+4(\cos 2\theta+1)\right)d\theta$	M1		Attempt to write $\cos^2\theta$ in terms of $\cos 2\theta$
	$=\left[22\theta+24\sin\theta+2\sin 2\theta\right]_{-\pi}^{\pi}$	A1F		correct integration ft wrong coefficients
	$=44\pi$	A1	6	CSO
(b)	At P, $r=4$; At Q, $r=2$;	B1		PI
	$P\ \{x=\}\ r\cos\theta=4\cos\frac{2\pi}{3}=-2$	M1		Attempt to use $r\cos\theta$
	$Q\ \{x=\}\ r\cos\theta=2\cos\pi=-2$	A1		Both
	Since P and Q have same *'x'*, PQ is vertical so QP is parallel to the vertical line $\theta=\frac{\pi}{2}$	E1	4	
(c)(i)	$OP=4$; $OS=8$;	B1		
	Angle $POS=\frac{\pi}{3}$	B1		or S $(4, 4\sqrt{3})$ and P $(-2, 2\sqrt{3})$
	$PS^2=4^2+8^2-2\times4\times8\times\cos\frac{\pi}{3}$ oe	M1		Cosine rule used in triangle POS OE $PS^2=(4+2)^2+(4\sqrt{3}-2\sqrt{3})^2$
	$PS=\sqrt{48}\ \ \{=4\sqrt{3}\}$	A1	4	
(ii)	Since $8^2=4^2+\left(\sqrt{48}\right)^2$, $OS^2=OP^2+PS^2\Rightarrow OPS$ is a right angle. (Converse of Pythagoras Theorem)	E1	1	Accept valid equivalents e.g. $PR=2PQ=2(2\sqrt{3})=PS$. $\angle SRP=\angle RSP=\angle RPO=\frac{\pi}{6}$ $\Rightarrow$ OPS is a right angle
	Total		**15**	
	TOTAL		**75**	