

General Certificate of Education
January 2007
Advanced Level Examination



MATHEMATICS
Unit Further Pure 2

MFP2

Thursday 1 February 2007 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
 - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Given that

$$4 \cosh^2 x = 7 \sinh x + 1$$

find the two possible values of $\sinh x$. (4 marks)

(b) Hence obtain the two possible values of x , giving your answers in the form $\ln p$.
(3 marks)

2 (a) Sketch on one diagram:

(i) the locus of points satisfying $|z - 4 + 2i| = 2$; (3 marks)

(ii) the locus of points satisfying $|z| = |z - 3 - 2i|$. (3 marks)

(b) Shade on your sketch the region in which

both $|z - 4 + 2i| \leq 2$

and $|z| \leq |z - 3 - 2i|$ (2 marks)

3 The cubic equation

$$z^3 + 2(1 - i)z^2 + 32(1 + i) = 0$$

has roots α , β and γ .

(a) It is given that α is of the form ki , where k is real. By substituting $z = ki$ into the equation, show that $k = 4$. (5 marks)

(b) Given that $\beta = -4$, find the value of γ . (2 marks)

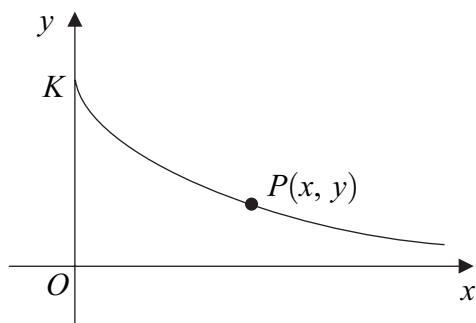
4 (a) Given that $y = \operatorname{sech} t$, show that:

$$(i) \quad \frac{dy}{dt} = -\operatorname{sech} t \tanh t; \quad (3 \text{ marks})$$

$$(ii) \quad \left(\frac{dy}{dt}\right)^2 = \operatorname{sech}^2 t - \operatorname{sech}^4 t. \quad (2 \text{ marks})$$

(b) The diagram shows a sketch of part of the curve given parametrically by

$$x = t - \tanh t \quad y = \operatorname{sech} t$$



The curve meets the y -axis at the point K , and $P(x, y)$ is a general point on the curve. The arc length KP is denoted by s . Show that:

$$(i) \quad \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \tanh^2 t; \quad (4 \text{ marks})$$

$$(ii) \quad s = \ln \cosh t; \quad (3 \text{ marks})$$

$$(iii) \quad y = e^{-s}. \quad (2 \text{ marks})$$

(c) The arc KP is rotated through 2π radians about the x -axis. Show that the surface area generated is

$$2\pi(1 - e^{-s}) \quad (4 \text{ marks})$$

Turn over for the next question

- 5 (a) Prove by induction that, if n is a positive integer,

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta \quad (5 \text{ marks})$$

- (b) Find the value of $\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)^6$. (2 marks)

- (c) Show that

$$(\cos \theta + i \sin \theta)(1 + \cos \theta - i \sin \theta) = 1 + \cos \theta + i \sin \theta \quad (3 \text{ marks})$$

- (d) Hence show that

$$\left(1 + \cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)^6 + \left(1 + \cos \frac{\pi}{6} - i \sin \frac{\pi}{6}\right)^6 = 0 \quad (4 \text{ marks})$$

- 6 (a) Find the three roots of $z^3 = 1$, giving the non-real roots in the form $e^{i\theta}$, where $-\pi < \theta \leq \pi$. (2 marks)

- (b) Given that ω is one of the non-real roots of $z^3 = 1$, show that

$$1 + \omega + \omega^2 = 0 \quad (2 \text{ marks})$$

- (c) By using the result in part (b), or otherwise, show that:

(i) $\frac{\omega}{\omega + 1} = -\frac{1}{\omega}$; (2 marks)

(ii) $\frac{\omega^2}{\omega^2 + 1} = -\omega$; (1 mark)

(iii) $\left(\frac{\omega}{\omega + 1}\right)^k + \left(\frac{\omega^2}{\omega^2 + 1}\right)^k = (-1)^k 2 \cos \frac{2}{3}k\pi$, where k is an integer. (5 marks)

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- 7 (a) Use the identity $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$ with $A = (r + 1)x$ and $B = rx$ to show that

$$\tan rx \tan(r + 1)x = \frac{\tan(r + 1)x}{\tan x} - \frac{\tan rx}{\tan x} - 1 \quad (4 \text{ marks})$$

- (b) Use the method of differences to show that

$$\tan \frac{\pi}{50} \tan \frac{2\pi}{50} + \tan \frac{2\pi}{50} \tan \frac{3\pi}{50} + \dots + \tan \frac{19\pi}{50} \tan \frac{20\pi}{50} = \frac{\tan \frac{2\pi}{5}}{\tan \frac{\pi}{50}} - 20 \quad (5 \text{ marks})$$

END OF QUESTIONS

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