

General Certificate of Education
January 2006
Advanced Level Examination



MATHEMATICS
Unit Further Pure 2

MFP2

Friday 27 January 2006 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Show that

$$\frac{1}{r^2} - \frac{1}{(r+1)^2} = \frac{2r+1}{r^2(r+1)^2} \quad (2 \text{ marks})$$

(b) Hence find the sum of the first n terms of the series

$$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \dots \quad (4 \text{ marks})$$

2 The cubic equation

$$x^3 + px^2 + qx + r = 0$$

where p , q and r are real, has roots α , β and γ .

(a) Given that

$$\alpha + \beta + \gamma = 4 \quad \text{and} \quad \alpha^2 + \beta^2 + \gamma^2 = 20$$

find the values of p and q . (5 marks)

(b) Given further that one root is $3 + i$, find the value of r . (5 marks)

3 The complex numbers z_1 and z_2 are given by

$$z_1 = \frac{1+i}{1-i} \quad \text{and} \quad z_2 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

(a) Show that $z_1 = i$. (2 marks)

(b) Show that $|z_1| = |z_2|$. (2 marks)

(c) Express both z_1 and z_2 in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. (3 marks)

(d) Draw an Argand diagram to show the points representing z_1 , z_2 and $z_1 + z_2$. (2 marks)

(e) Use your Argand diagram to show that

$$\tan \frac{5}{12}\pi = 2 + \sqrt{3} \quad (3 \text{ marks})$$

- 4 (a) Prove by induction that

$$2 + (3 \times 2) + (4 \times 2^2) + \dots + (n + 1) 2^{n-1} = n 2^n$$

for all integers $n \geq 1$.

(6 marks)

- (b) Show that

$$\sum_{r=n+1}^{2n} (r + 1) 2^{r-1} = n 2^n (2^{n+1} - 1) \quad (3 \text{ marks})$$

- 5 The complex number z satisfies the relation

$$|z + 4 - 4i| = 4$$

- (a) Sketch, on an Argand diagram, the locus of z .

(3 marks)

- (b) Show that the greatest value of $|z|$ is $4(\sqrt{2} + 1)$.

(3 marks)

- (c) Find the value of z for which

$$\arg(z + 4 - 4i) = \frac{1}{6}\pi$$

Give your answer in the form $a + ib$.

(3 marks)

Turn over for the next question

6 It is given that $z = e^{i\theta}$.

(a) (i) Show that

$$z + \frac{1}{z} = 2 \cos \theta \quad (2 \text{ marks})$$

(ii) Find a similar expression for

$$z^2 + \frac{1}{z^2} \quad (2 \text{ marks})$$

(iii) Hence show that

$$z^2 - z + 2 - \frac{1}{z} + \frac{1}{z^2} = 4 \cos^2 \theta - 2 \cos \theta \quad (3 \text{ marks})$$

(b) Hence solve the quartic equation

$$z^4 - z^3 + 2z^2 - z + 1 = 0$$

giving the roots in the form $a + ib$. (5 marks)

7 (a) Use the definitions

$$\sinh \theta = \frac{1}{2}(e^\theta - e^{-\theta}) \quad \text{and} \quad \cosh \theta = \frac{1}{2}(e^\theta + e^{-\theta})$$

to show that:

(i) $2 \sinh \theta \cosh \theta = \sinh 2\theta$; (2 marks)

(ii) $\cosh^2 \theta + \sinh^2 \theta = \cosh 2\theta$. (3 marks)

(b) A curve is given parametrically by

$$x = \cosh^3 \theta, \quad y = \sinh^3 \theta$$

(i) Show that

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = \frac{9}{4} \sinh^2 2\theta \cosh 2\theta \quad (6 \text{ marks})$$

(ii) Show that the length of the arc of the curve from the point where $\theta = 0$ to the point where $\theta = 1$ is

$$\frac{1}{2} \left[(\cosh 2)^{\frac{3}{2}} - 1 \right] \quad (6 \text{ marks})$$

END OF QUESTIONS