

General Certificate of Education
June 2005
Advanced Subsidiary Examination



MATHEMATICS
Unit Further Pure 1

MFP1

Wednesday 22 June 2005 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
- the **blue** AQA booklet of formulae and statistical tables;
- an insert for use in Question 7 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information

- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

(a) Calculate the matrices:

(i) $\mathbf{A} + \mathbf{B}$; (2 marks)

(ii) \mathbf{AB} . (2 marks)

(b) Show that $\mathbf{A} + \mathbf{B} - \mathbf{AB} = k\mathbf{I}$, where k is an integer and \mathbf{I} is the 2×2 identity matrix. (2 marks)

2 A curve satisfies the differential equation

$$\frac{dy}{dx} = \sin 2x$$

where the angle $2x$ is measured in **radians**.

Starting at the point $(0.5, 1)$ on the curve, use a step-by-step method with a step length of 0.1 to estimate the value of y at $x = 0.7$. Give your answer to three significant figures. (6 marks)

3 (a) Use the formulae

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$$

and

$$\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$$

to show that

$$\sum_{r=1}^n r^2(r-1) = \frac{1}{12}n(n^2-1)(3n+2) \quad (4 \text{ marks})$$

(b) Use the result from part (a) to find the value of

$$\sum_{r=4}^{11} r^2(r-1) \quad (3 \text{ marks})$$

4 The function f is defined for all real values of x by

$$f(x) = x^3 + x$$

(a) Express $f(2 + h) - f(2)$ in the form

$$ph + qh^2 + rh^3$$

where p , q and r are integers.

(5 marks)

(b) Use your answer to part (a) to find the value of $f'(2)$.

(2 marks)

5 Find the general solutions of the following equations, giving your answers in terms of π :

(a) $\tan 3x = \sqrt{3}$;

(3 marks)

(b) $\tan\left(3x - \frac{\pi}{3}\right) = -\sqrt{3}$.

(4 marks)

6 The equation

$$x^2 - 4x + 13 = 0$$

has roots α and β .

(a) (i) Write down the values of $\alpha + \beta$ and $\alpha\beta$.

(2 marks)

(ii) Deduce that $\alpha^2 + \beta^2 = -10$.

(2 marks)

(iii) Explain why the statement $\alpha^2 + \beta^2 = -10$ implies that α and β cannot both be real.
(2 marks)

(b) Find in the form $p + iq$ the values of:

(i) $(\alpha + i) + (\beta + i)$;

(1 mark)

(ii) $(\alpha + i)(\beta + i)$.

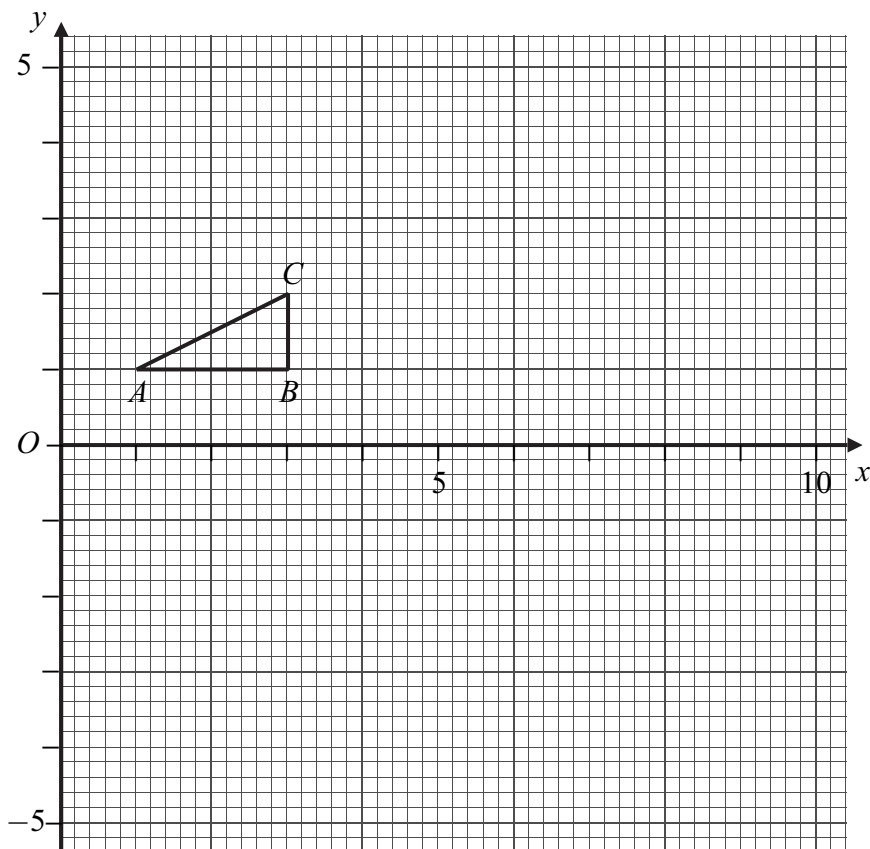
(2 marks)

(c) Hence find a quadratic equation with roots $(\alpha + i)$ and $(\beta + i)$.

(2 marks)

7 [Figure 1, printed on the insert, is provided for use in this question.]

The diagram shows a triangle with vertices $A(1, 1)$, $B(3, 1)$ and $C(3, 2)$.



- (a) The triangle DEF is obtained by applying to triangle ABC the transformation T represented by the matrix

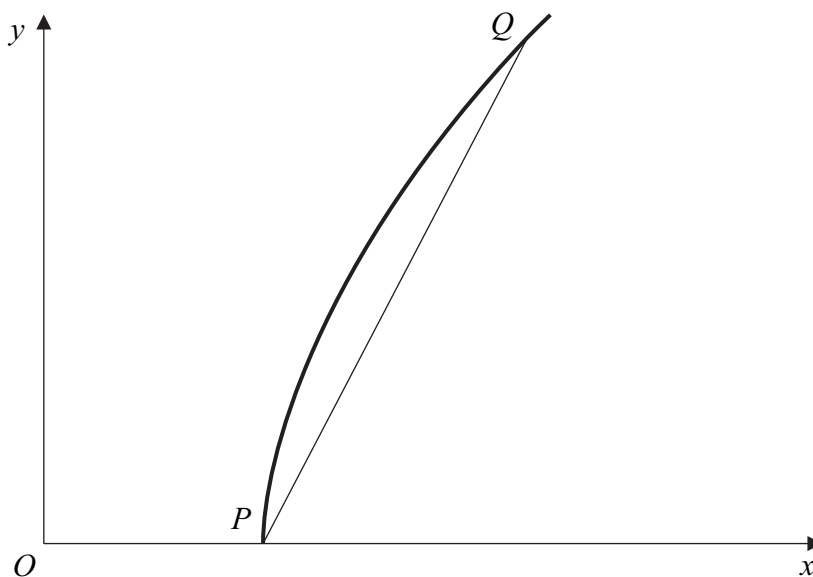
$$\begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix}$$

- (i) Calculate the coordinates of D , E and F . (4 marks)
- (ii) Draw the triangle DEF on **Figure 1**. (2 marks)
- (b) Given that T is a combination of an enlargement and a rotation, find the exact value of:
- (i) the scale factor of the enlargement; (2 marks)
- (ii) the magnitude of the angle of the rotation. (2 marks)

8 The diagram shows a part of the curve

$$\frac{x^2}{4} - \frac{y^2}{6} = 1$$

and a chord PQ of the curve, where P lies on the x -axis.



- (a) Write down the coordinates of P . (1 mark)
- (b) The gradient of the chord PQ is 2. Find the coordinates of Q . (7 marks)

9 The function f is defined by

$$f(x) = \frac{x^2 + 4x}{x^2 + 9}$$

- (a) (i) The graph of $y = f(x)$ has an asymptote which is parallel to the x -axis. Find the equation of this asymptote. (1 mark)
- (ii) Explain why the graph of $y = f(x)$ has no asymptotes parallel to the y -axis. (2 marks)
- (b) Show that the equation $f(x) = k$ has two equal roots if $9k^2 - 9k - 4 = 0$. (4 marks)
- (c) Hence find the coordinates of the two stationary points on the graph of $y = f(x)$. (6 marks)

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| Candidate Signature | | | | | | | | | | | |

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Insert for use in Question 7.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

TURN OVER FOR FIGURE 1

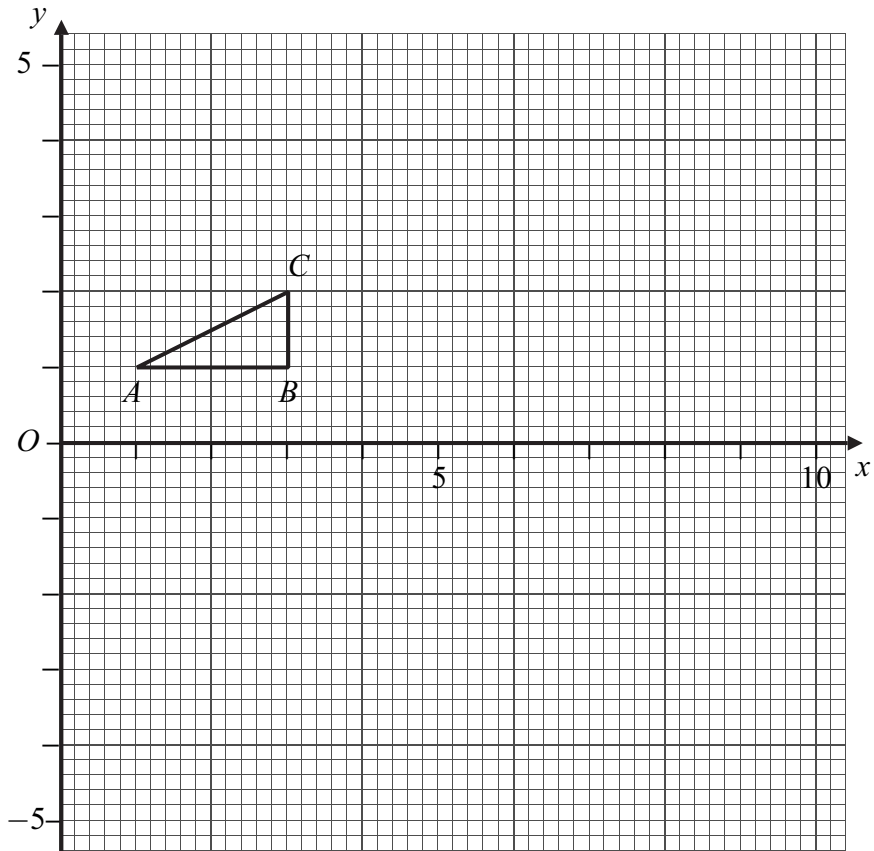


Figure 1 (for use in Question 7)