General Certificate of Education
January 2007
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Friday 26 January 2007 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Solve the following equations, giving each root in the form $a + bi$:

(i) $x^2 + 16 = 0$; *(2 marks)*

(ii) $x^2 - 2x + 17 = 0$. *(2 marks)*

(b) (i) Expand $(1 + x)^3$. *(2 marks)*

(ii) Express $(1 + i)^3$ in the form $a + bi$. *(2 marks)*

(iii) Hence, or otherwise, verify that $x = 1 + i$ satisfies the equation

$$x^3 + 2x - 4i = 0$$ *(2 marks)*

2 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

(a) Calculate:

(i) $\mathbf{A} + \mathbf{B}$; *(2 marks)*

(ii) $\mathbf{BA}$. *(3 marks)*

(b) Describe fully the geometrical transformation represented by each of the following matrices:

(i) $\mathbf{A}$; *(2 marks)*

(ii) $\mathbf{B}$; *(2 marks)*

(iii) $\mathbf{BA}$. *(2 marks)*

3 The quadratic equation

$$2x^2 + 4x + 3 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^2 + \beta^2 = 1$. *(3 marks)*

(c) Find the value of $\alpha^4 + \beta^4$. *(3 marks)*

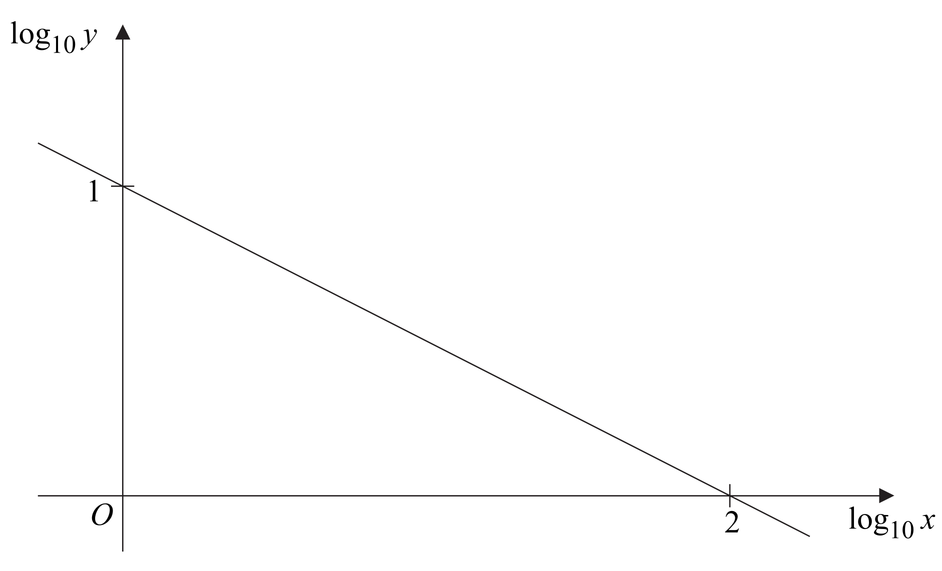
4 The variables x and y are related by an equation of the form

$$y = ax^b$$

where a and b are constants.

(a) Using logarithms to base 10, reduce the relation $y = ax^b$ to a linear law connecting $\log_{10} x$ and $\log_{10} y$. *(2 marks)*

(b) The diagram shows the linear graph that results from plotting $\log_{10} y$ against $\log_{10} x$.



Find the values of a and b. *(4 marks)*

5 A curve has equation

$$y = \frac{x}{x^2 - 1}$$

(a) Write down the equations of the three asymptotes to the curve. *(3 marks)*

(b) Sketch the curve.

(You are given that the curve has no stationary points.) *(4 marks)*

(c) Solve the inequality

$$\frac{x}{x^2 - 1} > 0$$ *(3 marks)*

6 (a) (i) Expand $(2r-1)^2$. *(1 mark)*

(ii) Hence show that

$$\sum_{r=1}^{n} (2r-1)^2 = \tfrac{1}{3}n(4n^2 - 1)$$ *(5 marks)*

(b) Hence find the sum of the squares of the odd numbers between 100 and 200. *(4 marks)*

7 The function f is defined for all real numbers by

$$\mathrm{f}(x) = \sin\left(x + \frac{\pi}{6}\right)$$

(a) Find the general solution of the equation $\mathrm{f}(x) = 0$. *(3 marks)*

(b) The quadratic function g is defined for all real numbers by

$$\mathrm{g}(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}x - \frac{1}{4}x^2$$

It can be shown that $\mathrm{g}(x)$ gives a good approximation to $\mathrm{f}(x)$ for small values of x.

(i) Show that $\mathrm{g}(0.05)$ and $\mathrm{f}(0.05)$ are identical when rounded to four decimal places. *(2 marks)*

(ii) A chord joins the points on the curve $y = \mathrm{g}(x)$ for which $x = 0$ and $x = h$. Find an expression in terms of h for the gradient of this chord. *(2 marks)*

(iii) Using your answer to part (b)(ii), find the value of $\mathrm{g}'(0)$. *(1 mark)*

8 A curve C has equation

$$\frac{x^2}{25} - \frac{y^2}{9} = 1$$

(a) Find the y-coordinates of the points on C for which $x = 10$, giving each answer in the form $k\sqrt{3}$, where k is an integer. *(3 marks)*

(b) Sketch the curve C, indicating the coordinates of any points where the curve intersects the coordinate axes. *(3 marks)*

(c) Write down the equation of the tangent to C at the point where C intersects the positive x-axis. *(1 mark)*

(d) (i) Show that, if the line $y = x - 4$ intersects C, the x-coordinates of the points of intersection must satisfy the equation

$$16x^2 - 200x + 625 = 0$$

(3 marks)

(ii) Solve this equation and hence state the relationship between the line $y = x - 4$ and the curve C. *(2 marks)*

END OF QUESTIONS

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