General Certificate of Education
January 2005
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Tuesday 1 February 2005 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the **blue** AQA booklet of formulae and statistical tables;
- an insert for use in Questions 7 and 8 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The equation

$$x^2 - 5x - 2 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Find the value of $\alpha^2\beta + \alpha\beta^2$. *(2 marks)*

(c) Find a quadratic equation which has roots

$$\alpha^2\beta \quad \text{and} \quad \alpha\beta^2$$ *(3 marks)*

2 A curve has equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

(a) Sketch the curve, showing the coordinates of the points of intersection with the coordinate axes. *(3 marks)*

(b) Calculate the y-coordinates of the points of intersection of the curve with the line $x = 1$. Give your answers in the form $p\sqrt{2}$, where p is a rational number. *(3 marks)*

(c) The curve is translated one unit in the positive x direction. Write down the equation of the curve after the translation. *(2 marks)*

3 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Write down, in terms of x and y, an expression for z^*, the complex conjugate of z. *(1 mark)*

(b) Find, in terms of x and y, the real and imaginary parts of

$$2z - \mathrm{i}z^*$$ *(2 marks)*

(c) Find the complex number z such that

$$2z - \mathrm{i}z^* = 3\mathrm{i}$$ *(3 marks)*

4 For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(a) $\int_2^\infty 8x^{-3}\,\mathrm{d}x$; *(3 marks)*

(b) $\int_2^\infty (8x^{-3}+1)\,\mathrm{d}x$; *(1 mark)*

(c) $\int_2^\infty 8x^{-3}(x+1)\,\mathrm{d}x$. *(3 marks)*

5 (a) The transformation T_1 is defined by the matrix

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Describe this transformation geometrically. *(2 marks)*

(b) The transformation T_2 is an anticlockwise rotation about the origin through an angle of 60°.

Find the matrix of the transformation T_2. Use surds in your answer where appropriate. *(3 marks)*

(c) Find the matrix of the transformation obtained by carrying out T_1 followed by T_2. *(3 marks)*

6 The angle x radians satisfies the equation

$$\cos\left(2x+\frac{\pi}{6}\right)=\frac{1}{\sqrt{2}}$$

(a) Find the general solution of this equation, giving the roots as exact values in terms of π. *(6 marks)*

(b) Find the **number** of roots of the equation which lie between 0 and 2π. *(2 marks)*

7 [**Figure 1**, printed on the insert, is provided for use in this question.]

The variables x and y are known to be related by an equation of the form

$$y^3 = ax^2 + b$$

where a and b are constants.

Experimental evidence has provided the following approximate values:

x	1.5	4.0	5.0	6.5	8.0
y	5.0	6.3	7.0	8.0	9.0

(a) On **Figure 1**, draw a linear graph connecting the variables X and Y, where

$$X = x^2 \quad \text{and} \quad Y = y^3$$ *(5 marks)*

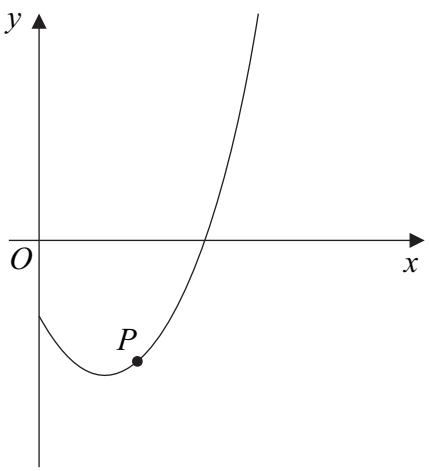
(b) From your graph, find approximate values for the constants a and b. *(3 marks)*

8 [**Figure 2**, printed on the insert, is provided for use in this question.]

The diagram shows a part of the graph of $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = x^3 - 2x - 1$$

The point P has coordinates $(1, -2)$.



(a) Taking $x_1 = 1$ as a first approximation to a root of the equation $\mathrm{f}(x) = 0$, use the Newton-Raphson method to find a second approximation, x_2, to the root. *(3 marks)*

(b) On **Figure 2**, draw a straight line to illustrate the Newton-Raphson method as used in part (a).

Mark x_1 and x_2 on **Figure 2**. *(2 marks)*

(c) By considering $\mathrm{f}(2)$, show that the second approximation found in part (a) is not as good as the first approximation. *(2 marks)*

(d) Taking $x_1 = 1.6$ as a first approximation to the root, use the Newton-Raphson method to find a second approximation to the root. Give your answer to three decimal places. *(2 marks)*

TURN OVER FOR THE NEXT QUESTION

9 The function f is defined by

$$f(x) = \frac{x^2 + 2x + 2}{x^2}$$

(a) Write down the equations of the two asymptotes to the curve $y = f(x)$. *(2 marks)*

(b) By considering the expression $x^2 + 2x + 2$:

(i) show that the graph of $y = f(x)$ does not intersect the x-axis; *(2 marks)*

(ii) find the non-real roots of the equation $f(x) = 0$. *(3 marks)*

(c) (i) Show that, if the equation $f(x) = k$ has two equal roots, then

$$4 - 8(1 - k) = 0$$ *(3 marks)*

(ii) Deduce that the graph of $y = f(x)$ has exactly one stationary point and find its coordinates. *(4 marks)*

END OF QUESTIONS

Surname		Other Names	
Centre Number						Candidate Number				
Candidate Signature										

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ASSESSMENT and QUALIFICATIONS ALLIANCE

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Insert for use in answering Questions 7 and 8.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

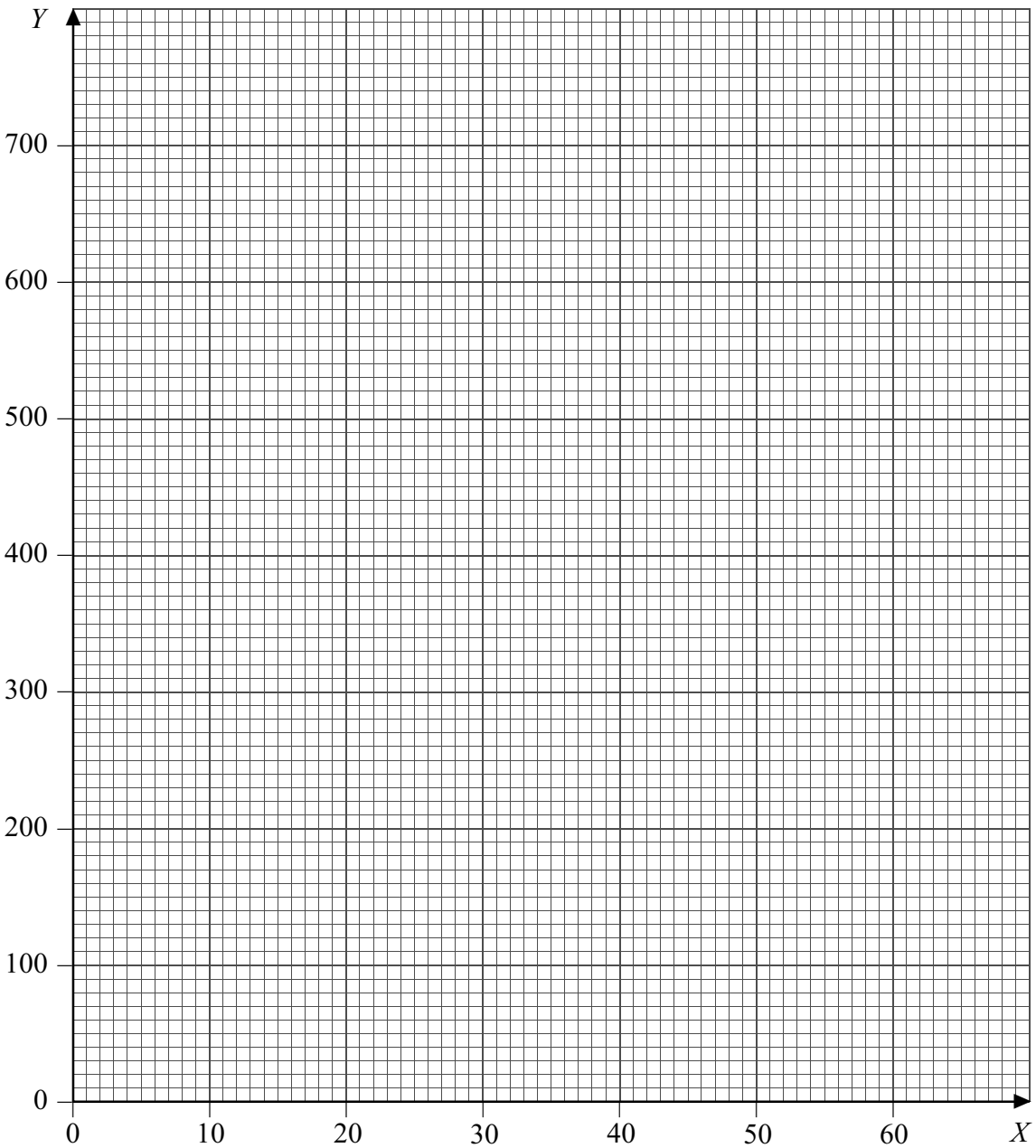

Figure 1 (for Question 7)

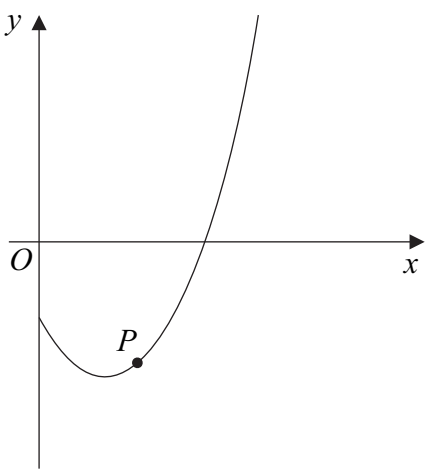

Figure 2 (for Question 8)

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