General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) **MBS7**
Unit Statistics 7

Monday 21 June 2004 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBS7.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The weight, in kilograms, of a particular make of paving brick may be assumed to be normally distributed with mean μ and variance σ^2.

A random sample of 10 bricks gave the following weights, in kilograms.

1.95 2.14 2.25 2.06 2.16 1.95 2.26 1.97 2.05 2.21

(a) Past experience suggests that $\sigma^2 = 0.01$. Verify that this value continues to be plausible by testing, at the 5% level of significance, the hypothesis that $\sigma^2 = 0.01$. *(7 marks)*

(b) Hence construct a 99% confidence interval for μ. *(5 marks)*

2 Between 8 am and 6 pm, the time, T minutes, between successive arrivals of vehicles for fuel at a village garage may be assumed to have an exponential distribution with a mean of 8.

(a) Write down the numerical value for the standard deviation of T. *(1 mark)*

(b) Calculate the probability that the time between successive arrivals of vehicles for fuel at the garage is between 5 minutes and 15 minutes. *(3 marks)*

(c) Between 6 pm and 9 pm, the time, S minutes, between successive arrivals of vehicles for fuel at the garage may be assumed to be independent of T and to have an exponential distribution with a mean of 15.

Calculate the probability that no vehicles arrive for fuel at the garage between 5.45 pm and 6.15 pm. *(5 marks)*

3 A manufacturer of 13-amp fuses states that 1.5 per cent of fuses produced are faulty.

(a) A wholesaler, who suspects that more than 1.5 per cent of fuses produced are faulty, tests a random sample of 100 fuses and finds that 4 fuses are faulty.

(i) Write down null and alternative hypotheses for investigating the wholesaler's suspicion. *(2 marks)*

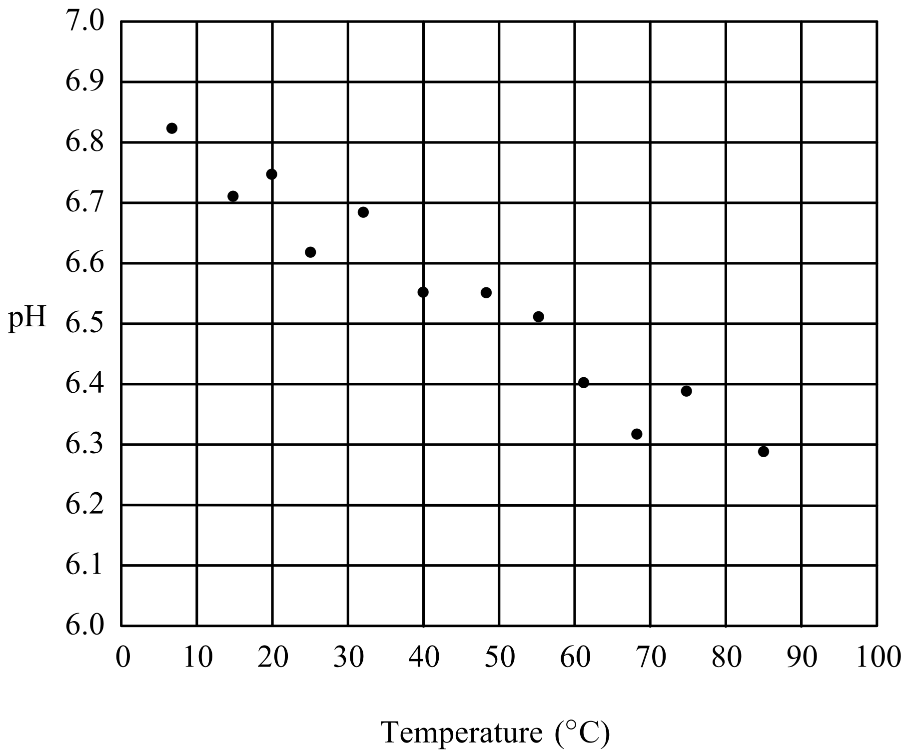
(ii) Use a Poisson approximation and the 10% level of significance to investigate the wholesaler's suspicion. *(6 marks)*

(b) In view of the wholesaler's suspicion, the manufacturer decides to examine a random sample of 2000 fuses and finds that 36 fuses are faulty.

Use this information to investigate, again at the 10% level of significance, the wholesaler's suspicion. *(5 marks)*

4 The following scatter diagram illustrates the data collected as part of a study into the effect of temperature, $x\,°$C, on the pH value, y, of a particular liquid.

Scatter Diagram of pH versus Temperature



(a) Explain what this diagram reveals about the relationship between temperature and pH. *(2 marks)*

(b) Given the following information, determine the equation of the least squares regression line, $y = \hat{\alpha} + \hat{\beta}x$.

$$\bar{x} = 44 \qquad \bar{y} = 6.55 \qquad S_{xx} = 6938 \qquad S_{xy} = -46.5$$ *(2 marks)*

(c) The relationship between x and y may be modelled by

$$y = \alpha + \beta x + \varepsilon \quad \text{where} \quad \varepsilon \sim \mathrm{N}(0, \sigma^2).$$

(i) Given that $n = 12$ and $S_{yy} = 0.3268$, calculate an unbiased estimate of σ^2. *(2 marks)*

(ii) Investigate, at the 1% level of significance, the hypothesis that $\beta < -0.005$. *(7 marks)*

(iii) Explain, in context, your conclusion to part (c)(ii). *(2 marks)*

5 (a) The random variable X has a normal distribution with mean 200 and variance 100.

(i) Show that $\mathrm{P}(X < x) = \mathrm{P}(Z < (0.1x - 20))$ where $Z \sim \mathrm{N}(0, 1)$. *(1 mark)*

(ii) **Table A** shows the probability that a value, x, lies within a particular interval.

Table A

Interval	Probability
$x < 180$	
$180 \leqslant x < 190$	
$190 \leqslant x < 200$	0.34134
$200 \leqslant x < 210$	
$210 \leqslant x < 220$	
$x \geqslant 220$	

Copy and complete this table. *(2 marks)*

(b) **Table B** shows the frequency distribution of the times, X minutes, taken to complete a post round on a sample of 600 days.

Table B

Time to complete post round	Frequency
$x < 180$	19
$180 \leqslant x < 190$	74
$190 \leqslant x < 200$	193
$200 \leqslant x < 210$	218
$210 \leqslant x < 220$	85
$x \geqslant 220$	11

Using a goodness of fit test and the 5% level of significance, test the hypothesis that X may be modelled by a normal distribution with mean 200 and variance 100.
(Calculate expected frequencies to two decimal places.) *(8 marks)*

END OF QUESTIONS