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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics and Statistics 6320
Specification B

MBS7 Statistics 7

Mark Scheme

2005 examination – June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
√or ft or F		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
–*x* ee		deduct *x* marks for each error
pi		possibly implied
sca		substantially correct approach

Abbreviations used in Marking

MC – *x*	deducted *x* marks for mis-copy
MR – *x*	deducted *x* marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

Application of Mark Scheme

No method shown:	
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

Mathematics and Statistics B Statistics 7 MBS7 June 2005

Q	Solution	Marks	Total	Comments
1	$\sum x = 2000 \quad \sum x^2 = 400034$			
	$s_{n-1}^2 = \mathbf{3.78} \quad s_{n-1} = \mathbf{1.94}$ $S_{xx} = \sum (x - \bar{x})^2 = \mathbf{34}$ $s_n^2 = \mathbf{3.4} \quad s_n = \mathbf{1.84}$	B1		Any one; awrt
	Degrees of freedom, $\nu = 10 - 1 = \mathbf{9}$	B1		cao
	99% $\Rightarrow$ 0.005 and 0.995, so values are **1.73 to 1.74** and **23.5 to 23.6**	B1		**Both** awfw; (1.735, 23.589)
	CI for σ^2 is $\dfrac{(n-1)s^2}{\chi^2(\mathrm{U})}$ & $\dfrac{(n-1)s^2}{\chi^2(\mathrm{L})}$	M1		Use of; allow mixture of $(n-1)$ & n, s_{n-1}^2 & s_n^2
	CI for σ^2 is $\dfrac{34}{23.589}$ & $\dfrac{34}{1.735}$	A1√		ft on χ^2-values Allow mixture of $(n-1)$ & n, s_{n-1}^2 & s_n^2
	CI for σ is $\left(\sqrt{1.44}, \sqrt{19.6}\right)$	m1		$\sqrt{}$ of **two positive** values
	CI for σ is **(1.2, 4.4)**	A1	7	awrt
		Total	**7**	

MBS7 (cont)

Q	Solution	Marks	Total	Comments
2	H_0: $\sigma_1^2 = \sigma_2^2$ H_1: $\sigma_1^2 \neq \sigma_2^2$	B1		**Both**; must be population
	SL $\alpha = 0.05$ (5%) DF $\nu_1 = \mathbf{11}$ $\nu_2 = \mathbf{9}$	B1		**Both** cao; may be reversed
	CV $F = \mathbf{3.91}$ or CV $F = (3.588)^{-1} = \mathbf{0.28}$	B1		(3.912) **Either** awrt (0.279)
	$F = \dfrac{\text{unbiased estimate}}{\text{unbiased estimate}}$	M1		Use of
	$= \dfrac{20.2}{4.3} = \mathbf{4.7}$ or $= \dfrac{4.3}{20.2} = \mathbf{0.21}$	A1		**Either** awrt
	Thus, at 5% level, reject H_0, so **evidence of a difference** in variability	A1	6	**Must compare** 4.7 with 3.9 **or compare** 0.21 with 0.28
	Note Confusing DF, CV and *F*-ratio is likely to score a maximum of 4 marks: B1 B1 B0 M1 A1 A0			
		Total	**6**	

MBS7 (cont)

Q	Solution	Marks	Total	Comments
3				
(a)	Graph shows a **linear** relationship	B1		Or equivalent
	Conditions suggest only other influence is **random variation**	B1	2	Or equivalent
(b)	$\hat{\beta} = \frac{-328}{1500}$ = **–0.219 to –0.218**	B1		awfw (0.21866)
		(B1)		If **both** scored in (d)
	$\hat{\alpha} = 9.1 - \hat{\beta} \times 30$ = **15.6 to 15.7**	B1	2	awfw (15.66)
(c)	$s^2 = \frac{1}{n-2}\left(S_{yy} - \frac{(S_{xy})^2}{S_{xx}}\right)$	M1		Use of; or equivalent
		(B1)		If scored in (d)
	= **0.116 to 0.117**	A1	2	awfw (0.11676)
(d)	H_0: $\beta = -0.2$	B1		**Both**; must be population
	H_1: $\beta \neq -0.2$			
	SL $\alpha = 0.05$ (5%) DF $\nu = 9 - 2 = \mathbf{7}$	B1		cao
	CV $\lvert t \rvert$ = **2.36 to 2.37**	B1		awfw; ignore sign (2.2365)
	$t = \frac{\hat{\beta} - \beta_0}{\sqrt{s^2/S_{xx}}} = \frac{(-0.21866) - (-0.2)}{\sqrt{0.11676/1500}}$	M1		Use of; allow $\beta_0 = 0$
	= **–2.17 to –2.03**	A1		awfw; ignore sign (–2.116)
	Thus, at 5% level, accept H_0, so **no evidence to reject null hypothesis** that $\beta = -0.2$	A1√	6	ft on *t*-value and CV but signs must be consistent
(e)	For each **1 or 10** mg **increase/rise/change** in additive	B1		Or equivalent
	there is an average **decrease/fall/change** in drying time of **0.2 or 2** hours	B1	2	Or equivalent
	Total		**14**	

MBS7 (cont)

Q	Solution	Marks	Total	Comments
4				
	H_0: average per hour **is constant**	B1		Allow rectangular or uniform
	H_1: not H_0			At least H_0
	SL $\alpha = 0.10$ (10%)			
	DF $\nu = \mathbf{4}$	B1		cao
	CV $\chi^2 = \mathbf{7.77}$ **to** $\mathbf{7.78}$			awfw (7.779)
	or	B1		
	CV $\chi^2 = \mathbf{6.25}$			awrt (6.251)
	Estimate of average/hour = $\frac{480}{32} = \mathbf{15}$	B1		cao; may be implied by E
	Day / O / E: Mon 136 **112.5**; Tues 43 **52.5**; Wed 89 **112.5**; Thur 127 **135.0**; Fri 85 **67.5**; Week 480 480.0	M1		Use of hours
		(B1)		cao all 5 E = 96
		A1		cao all 5 E
	$\chi^2 = \sum \frac{(O-E)^2}{E}$	M1		Use of; even for all 5 E = 96 but O-values must be customers
	= **16.5 to 16.6**	A1		awfw (16.548) (All 5 E = 96 gives 57.7)
	Thus, at 10% level, reject H_0, so **evidence** that average per hour is **not constant**	A1√	9	ft on χ^2-value & **upper tail** CV but E-values must be correct or = 96
		Total	**9**	

MBS7 (cont)

Q	Solution	Marks	Total	Comments
5	$C \sim N(1085, 18^2)$ $B \sim N(420, 6^2)$			
(a)				
(i)	$W = B + \Sigma C$			
	Mean, $\mu_W = 420 + 6 \times 1085 =$ **6930**	B1		cao; accept 6.93
	Variance, $\sigma_W^2 = 6^2 + 6 \times 18^2$	M1		Adding (7) variances
	= **1980**	A1	3	cao; accept 0.00198 (SD = 44.5 or 0.0445 $\Rightarrow$ A0)
(ii)	$P(W < 7000) = P\left(Z < \frac{7000 - 6930}{\sqrt{1980}}\right)$	M1		Standardising (7000 or 7) using consistent units and ft (μ & σ), not σ^2
	= $P(Z < 1.57)$ = **0.941 to 0.943**	A1	2	awfw (0.94179)
(b)	$D = C_1 - C_2$			
	Mean, μ_D = **0**	B1		cao; may be implied in *z*-value
	Variance, $\sigma_D^2 = 18^2 + 18^2 =$ **648**	B1		cao
	or SD, σ_D = **25.4 to 25.5**			awfw
	$P(D > 50) = P\left(Z > \frac{50 - 0}{\sqrt{648}}\right)$	M1		Standardising (50) using ft (μ & σ), not σ^2 Allow (0 – 50) or (–50)
	= $P(Z > 1.96) = 1 - \Phi(1.96)$			
	= **0.024 to 0.025**	A1		awfw
	$P(\lvert D \rvert > 50) = 2 \times P(D > 50)$	M1		Use of multiplier of 2
	= **0.05**	A1√	6	ft on $0 < P(D > 50) < 0.5$
		Total	**11**	

MBS7 (cont)

Q	Solution	Marks	Total	Comments
6				
(a)	H_0: $p = 0.4$ H_1: $p > 0.4$	B1		**Both**; <u>can be scored in (b)</u>
	$P(X \geq 15 \mid n = 30, p = 0.4)$	M1		Use of B(30, 0.4)
	$= \mathbf{1 - P(X \leq 14 \text{ or } 15)}$	m1		Use of; or $P(X < CV) > 0.9$
	$= 1 - 0.8246 =$ **0.175 to 0.176**	A1		awfw; or CV = 16
	Thus, at 10% level, accept H_0, so **no evidence to support company's claim**	A1√	5	ft on *p*-value and 0.10 (10%) or on 15 and CV
(b)	Normal approx with mean, $\mu =$ **60** or **0.4**	B1		**Either** cao
	and variance, $\sigma^2 =$ **36** or SD, $\sigma =$ **6** or variance, $\sigma^2 =$ **0.0016** or SD, $\sigma =$ **0.04**	B1		**One** cao
	CV $z =$ **1.28**	B1		awrt (1.2816)
	$z = \dfrac{x-\mu}{\sqrt{\sigma^2}} = \dfrac{(72.5 \text{ or } 73 \text{ or } 73.5) - 60}{\sqrt{36}}$	M1		Standardising (72, 72.5, 73, or 73.5) using ft (μ & σ), not σ^2 Or equivalents for $\hat{p}$
	$=$ **2.08 to 2.17** $\Rightarrow$ *p*-value = **0.015 to 0.019**	A1 (A1)		awfw Excludes use of 72 or 73.5
	Thus, at 10% level, reject H_0, so **evidence to support company's claim**	A1√	6	ft on *z*-value and CV or on *p*-value and 10%
	Note B(150, 0.4) $\Rightarrow$ 0.0193 < 0.10 $\Rightarrow H_1$ B(150, 0.4) $\Rightarrow$ 0.0128 < 0.10 $\Rightarrow H_1$ B(150, 0.4) $\Rightarrow$ 0.98 to 0.989 > 0.10 $\Rightarrow H_0$			M2 A3 A1√ M2 A2 A1√ M1 A2 A0√
(c)	Early arrivals unlikely to affect other arrivals **or** Late arrivals likely to affect other arrivals **so** Likely to be valid **or** Unlikely to be valid	B1 B1	2	Very early arrivals may affect other possible early arrivals depending on previous
	Total		**13**	
	TOTAL		**60**	