

General Certificate of Education  
January 2004  
Advanced Level Examination



**MATHEMATICS AND STATISTICS  
(SPECIFICATION B)  
Unit Statistics 5**

**MBS5**

Wednesday 21 January 2004 Afternoon Session

**In addition to this paper you will require:**

- a 12-page answer book;
- the AQA booklet of formulae and statistical tables;
- one sheet of graph paper for use in Question 5;
- a ruler.

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBS5.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

**Information**

- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

**Advice**

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

---

Answer **all** questions.

---

- 1 A coin-operated machine dispenses soft drinks into cups when a button is pressed.
- (a) After the machine has been overhauled the amount,  $X$  millilitres, of soft drink dispensed on each press of the button may be modelled by a normal distribution with mean 490 and standard deviation 10.
- (i) Find the probability that  $X$  lies between 475 and 510. *(5 marks)*
- (ii) Find the value of  $X$  which will be exceeded on 97% of presses of the button. *(4 marks)*
- (b) The owner of the machine receives complaints that some cups are underfilled and suspects that the machine is due for an overhaul. Investigation shows that the amount,  $Y$  millilitres, of soft drink dispensed is less than 475 on 15% of presses of the button.
- Find the mean of  $Y$ , assuming that it may be modelled by a normal distribution with standard deviation 10. *(4 marks)*

- 2 A hospital kitchen buys oranges in large batches. In each batch, the vitamin C content of the flesh of the oranges, in milligrams per 10 grams, may be modelled by a normal distribution with standard deviation 0.11.

The vitamin C contents of the flesh of a random sample of 8 oranges from a particular batch were measured, with the following results:

1.32    1.11    1.39    1.22    1.25    1.57    1.42    1.36

- (a) Calculate a 90% confidence interval for the mean vitamin C content of the flesh of oranges in this batch. *(5 marks)*
- (b) A nutritionist, who draws up diets for individual patients, assumes that the flesh of oranges has a vitamin C content of at least 1.20. Comment on this assumption as it relates to this batch of oranges. *(2 marks)*
- (c) The nutritionist requests that a 95% confidence interval of width not more than 0.03 should be calculated for the mean vitamin C content of the flesh of oranges in each batch. Calculate how large a random sample would have to be taken from each batch in order to achieve this. *(5 marks)*

- 3 A make of car battery is advertised as having a mean lifetime of at least 40 months. A car magazine suspects that the mean lifetime is less than 40 months and so monitors the lifetimes, in months, of a random sample of 12 batteries, with the following results:

44 32 46 39 38 30 35 40 38 41 34 36

- (a) Assuming that lifetimes are normally distributed with standard deviation 5 months, investigate the magazine's suspicion, using the 5% significance level. *(8 marks)*
- (b) A larger random sample of 160 car batteries is now monitored. The lifetimes are found to have a mean of 39.2 months and a standard deviation of 4.2 months. Use this information to investigate the magazine's suspicion, using the 5% significance level. *(3 marks)*
- (c) State, with an explanation, which, if either, of the tests in parts (a) and (b) has the larger risk of:
- (i) a Type I error;
  - (ii) a Type II error if the mean lifetime is 40 months;
  - (iii) a Type II error if the mean lifetime is 39 months. *(6 marks)*

**TURN OVER FOR THE NEXT QUESTION**

- 4 A newspaper published an article concerning a proposed European law which would force motorists to pay compensation and damages in all accidents with cyclists, regardless of who is to blame. The newspaper received 150 letters in response to the article. The following table shows the number of letters received, classified by the writers' usual forms of transport and their attitudes to the proposal.

		Attitude to the proposal		
		For	Against	Neutral
Usual form of transport	Car	3	67	9
	Bicycle	27	5	10
	Other	8	12	9

- (a) The editor selects a letter at random to consider for publication. Find the probability that the letter selected is:
- for the proposal;
  - for the proposal, given that the writer usually travels by car. *(3 marks)*
- (b) The editor selects three letters at random, without replacement. Find the probability that one letter is for the proposal, one against and one neutral. *(4 marks)*
- (c) The proportions of letters suitable for publication were:
- 0.50 of those which were for the proposal;  
0.25 of those which were against the proposal;  
0.75 of those which were neutral.
- Find the probability that a randomly selected letter is for the proposal and suitable for publication. *(2 marks)*
  - Find the probability that a randomly selected letter is suitable for publication. *(3 marks)*
  - Find the probability that at least three letters will have to be selected (at random, without replacement) before a letter is found which is suitable for publication from a writer who is for the proposal. *(3 marks)*

5 [A sheet of graph paper is provided for use in this question.]

Hamish organises a house-to-house collection in aid of a refugee charity. Volunteers distribute leaflets and envelopes to houses in a designated area. The leaflet explains the aims of the charity and asks the householder to place a donation in the envelope. The volunteer will call back a few days later to collect the envelope.

The following table shows, for 10 volunteers, the number of houses,  $x$ , to which they delivered envelopes and the amount to the nearest £,  $y$ , they collected.

Volunteer	A	B	C	D	E	F	G	H	I	J
$x$	78	162	93	214	188	54	312	80	244	129
$y$	23	78	19	77	104	33	144	17	81	52

- (a) Plot a scatter diagram of the data. (3 marks)
- (b) Calculate the equation of the regression line of  $y$  on  $x$  and draw the line on your scatter diagram. (5 marks)
- (c) The following table shows the residuals for some of the volunteers.

Volunteer	A	B	C	D	E	F	G	H	I	J
Residual			-15.1	-12.7	26.2	16.8	9.2	-11.1	-22.5	1.3

- (i) Calculate the residuals for volunteers A and B. (3 marks)
- (ii) Calculate the mean **magnitude** of the 10 residuals. (2 marks)
- (d) Bryn, a new volunteer, delivered leaflets to 112 houses and collected £31. Advise Hamish whether Bryn should have been supervised when collecting the envelopes. Your answer should be supported with numerical evidence. (3 marks)
- (e) (i) If  $y = a + bx$  is the regression equation for a set of points  $(x_i, y_i)$ ,

$$\text{then } \bar{y} = a + b\bar{x}.$$

Explain why this fact ensures that the mean of the residuals for the set of points will be zero. (3 marks)

- (ii) When the point (112, 31), corresponding to Bryn's contribution, is included in the calculation of the regression equation in part (b), the table of residuals is as follows.

Volunteer	A	B	C	D	E	F	G	H	I	J	Bryn
Residual	-2.6	13.2	-13.6	-12.1	27.0	18.6	9.1	-9.5	-22.1	2.6	

Calculate the residual for Bryn. (2 marks)

- (iii) State how, if at all, the result of your calculation in part (e)(ii) would affect your answer to part (d). (2 marks)