

General Certificate of Education
June 2005
Advanced Level Examination



**MATHEMATICS AND STATISTICS
(SPECIFICATION B)
Unit Statistics 4**

MBS4

Thursday 16 June 2005 Afternoon Session

In addition to this paper you will require:

- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBS4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 The dissolved oxygen contents, in parts per million, of samples of water taken from a river were as follows:

4.9 3.6 4.3 4.2 3.7 5.0 3.5 3.9 4.4

The data may be regarded as a random sample from a normal distribution.

Calculate a 95% confidence interval for the mean dissolved oxygen content of water in the river.
(7 marks)

- 2 A political party held a conference, attended by delegates from local branches, to choose a new leader. The voting at the conference was carried out in public. In a confidential survey, randomly selected delegates were asked whether they would have voted differently if voting had been by secret ballot. The answers, classified by the ages of the delegates, are summarised in the following table.

	Definitely	Possibly	Definitely not
Under 65 years	4	12	6
65 years and over	10	28	54

- (a) Use a χ^2 distribution and the 5% significance level to examine whether the answer to the question is associated with the age of the delegate. (10 marks)
- (b) Interpret your result in part (a) in the context of the question. (1 mark)
- (c) Make a further comment about the ages of the delegates. (1 mark)

- 3 The weekly number of visits Wendell makes to the cinema may be modelled by the discrete random variable, Y , with the following probability distribution.

y	$P(Y = y)$
0	0.58
1	0.27
2	0.10
3	0.05

(a) Find:

- (i) the mean of Y ;
- (ii) the standard deviation of Y ;
- (iii) $E(Y^3)$. *(7 marks)*

(b) Write down:

- (i) the mode of Y ;
- (ii) the median of Y . *(2 marks)*

(c) Entry to the cinema costs Wendell £4.50 per visit. An annual (52 week) ticket is available for £120.

- (i) Which measure of average – mean, mode or median – should Wendell consider when deciding whether or not to buy an annual ticket? Explain your answer. *(2 marks)*
- (ii) Give **one** numerical argument in favour of Wendell buying an annual ticket. *(2 marks)*
- (iii) Give **one** disadvantage of buying an annual ticket. *(1 mark)*

4 While examining a water company's finances, an auditor selected a random sample of 90 customers, who owed the company money, in order to scrutinise their accounts. The amounts owed by these 90 customers had a mean of £197 and a standard deviation of £103.

(a) For customers who owed the company money:

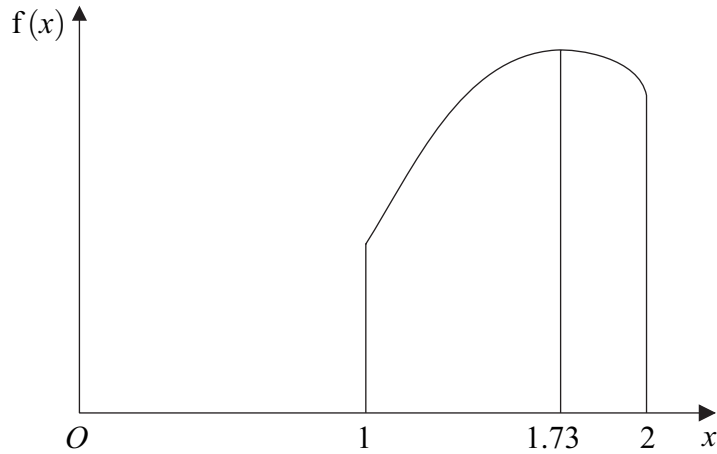
- (i) calculate a 95% confidence interval for the mean amount owed; *(4 marks)*
- (ii) state the width of the confidence interval that you have calculated in part (a)(i); *(1 mark)*
- (iii) find the percentage which would be associated with a confidence interval of width £30 calculated from the given data; *(5 marks)*
- (iv) find, approximately, the size of sample which would be required in order to calculate a 99% confidence interval of width £30 for the mean amount owed. *(4 marks)*

(b) Explain why:

- (i) your calculations in part (a) are valid whether or not the amounts owed may be modelled by a normal distribution; *(2 marks)*
- (ii) a normal distribution will **not** provide a good model for the amounts owed. *(2 marks)*

- 5 The sketch below shows the probability density function $f(x)$ of the continuous random variable X .

($f(x) = 0$ outside the interval $1 \leq x \leq 2$.)



You are given that

$$\int_1^{1.56} f(x) dx = 0.5 \qquad \int_{1.73}^2 f(x) dx = 0.291$$

$$\int_1^2 x f(x) dx = 1.54 \qquad \int_1^2 (x - 1.54)^2 f(x) dx = 0.0751$$

- (a) Write down the value of:

(i) $\int_1^2 f(x) dx$;

(ii) the mean of X ;

(iii) the median of X ;

(iv) the mode of X ;

(v) the range of X .

(5 marks)

- (b) Find:

(i) the probability that X lies between the median and the mode;

(2 marks)

(ii) the standard deviation of X ;

(2 marks)

(iii) $E(X^2)$.

(2 marks)

- 6 Santos, a newsagent, employs schoolchildren to deliver newspapers. Marilyn is to start delivering morning newspapers. She wishes to complete her delivery round early enough to ensure that she will not be late for school. Santos tells her that the average time taken to complete a delivery round is 40 minutes. Marilyn's friend Lula, who already delivers morning papers for Santos, says that the average time taken to complete a delivery round is 50 minutes.

Before starting her employment, Marilyn checks the times taken by 8 schoolchildren, employed by Santos, to complete their rounds. The times, in minutes, may be regarded as a random sample from a normal distribution and were as follows:

52 30 41 49 61 33 52 54

- (a) (i) By carrying out a hypothesis test, using the 5% significance level, show that the null hypothesis that the mean time to complete a round is equal to 40 minutes is accepted.
- (ii) By carrying out a hypothesis test, using the 5% significance level, show that the null hypothesis that the mean time to complete a round is equal to 50 minutes is accepted.
(12 marks)

- (b) The following three claims refer to the two tests carried out in part (a).

Claim 1 A Type I error has occurred in at least one of the tests.

Claim 2 A Type II error has occurred in both tests.

Claim 3 A Type II error has occurred in at least one of the tests.

For **each** of the three claims, state whether it is:

- A definitely true;
B possibly true;
C definitely not true.

Explain your answers.

(6 marks)

END OF QUESTIONS

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