

GCE 2004

June Series



Mark Scheme

Mathematics and Statistics B

MBS4

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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Dr Michael Cresswell Director General

Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
✓ or ft or F		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
-x ee		deduct x marks for each error
pi		possibly implied
sca		substantially correct approach

Abbreviations used in Marking

MC – x	deducted x marks for mis-copy
MR – x	deducted x marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

Application of Mark Scheme

No method shown:

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

More than one method / choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

Crossed out work	do not mark unless it has not been replaced
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Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate
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Mathematics and Statistics B Statistics 4 MBS4 June 2004

Question Number and Part	Solution	Marks	Total	Comments
1(a)	$E(X) = 0 \times 0.08 + 1 \times 0.30 + 2 \times 0.34 + 3 \times 0.15 + 4 \times 0.10 + 5 \times 0.03 = 1.98$	M1 A1	6	Method 1.98 cao
	$E(X^2) = 0 \times 0.08 + 1 \times 0.30 + 4 \times 0.34 + 9 \times 0.15 + 16 \times 0.10 + 25 \times 0.03 = 5.36$	M1		Method for $E(X^2)$ - may be implied
	$V(X) = 5.36 - 1.98^2 = 1.4396$	m1		Method for variance - disallow if called standard deviation
	$s.d = \sqrt{1.4396} = 1.20$	m1 A1		Completely correct method 1.20 (1.195 to 1.205) sc m0 A1 for variance 1.44 (1.435 to 1.445)
(b)(i)	mean $\frac{1.98}{280}$ s.d. $\frac{\sqrt{1.4396}}{280} = 0.0717$	B1✓ M1 A1	3	1.98 - their mean Method, their s.d. 0.0717 (0.0716 to 0.0718)
(ii)	$k = 1.98 + 2.3263 \times 0.071704 = 2.15$	B1 M1 A1	3	2.3263 (2.32 to 2.33) (their z) × (their s.d.) 2.15 (2.14 to 2.15)
(iii)	$280 \times 2.1468 = 601$	B1	1	600, 601, 602
	Total		13	
2(a)	$H_0: \mu = 200$ $H_1: \mu > 200$ $\bar{x} = 205.545$ $s = 7.2438$ $t = \frac{205.545 - 200}{\frac{7.2438}{\sqrt{11}}} = 2.54$ cv t_{10} for 5% 1-sided risk 1.812 Reject H_0 Conclude mean time to spoilage is greater than 200 hours.	B1 B1 B1 B1 M1 m1 A1 B1 B1✓ A1✓	10	One correct hypothesis - generous Both hypotheses correct - ungenerous 205.545 (205.5 to 205.6) 7.24 (7.24 to 7.25) - may be implied by correct t -statistic Use of their $\frac{s}{\sqrt{11}}$ Correct method for t - ignore sign 2.54 (2.535 - 2.545) 10 df 1.812 - allow 1.81, their df, disallow ± 1.812 Correct conclusion - must be compared with correct tail of t
(b)	No problem unless the 3 containers sold could not be treated as a random sample. Unlikely to bias sample.	E1 E1	2	Comment on randomness of 3 containers Correct deduction Allow comment on reduced sample size
(c)	Claiming mean time to spoilage is more than 200 hours when it isn't	E1 E1	2	Idea of Type 1 error In context - must be 1-sided
(d)(i)	0.05	B1		0.05 cao
(ii)	0	B2	3	0 cao
	Total		17	

MBS4 (cont)

Question Number and Part	Solution	Marks	Total	Comments
3(a)	$\bar{x} = 135.6$ $s = 0.45552$ 95% confidence interval for mean is $135.6 \pm 2.306 \times \frac{0.45552}{\sqrt{9}}$ i.e. 135.6 ± 0.350 135.25 to 135.95	B1 B1 B1 B1 ✓ M1 m1 B1 A1	8	135.6 cao 0.45552 (0.455 to 0.456) may be implied by correct interval 8df 2.306 Use of their $\frac{s}{\sqrt{9}}$ Correct method for interval Answer given to 4,5 or 6 sf, must be an interval 135.25 (135.2 to 135.3) and 135.95 (135.9 to 136) Or 135.6 cao ± 0.35 (0.349 to 0.351)
(b)	$135.8 \pm 1.96 \times \frac{3.9}{\sqrt{60}}$ 135.8 ± 0.987 134.81 to 136.79	B1 M1 m1 A1	4	1.96 cao Use of $\frac{3.9}{\sqrt{60}}$ Correct method for interval 134.81 (134.8 to 134.820) and 136.79 (136.78 to 136.8) Or 135.8 cao ± 0.987 (0.98 to 1)
(c)(i)	Decrease	B1	1	Decrease
(ii)	Increase - narrower interval → more likely to identify need for overhaul if $\mu \neq 135.0$	E1 E1	2	Increase Reason
(d)	Large variability as in (b) will lead to unsatisfactory production even if mean on target. Might prefer small variability as in (a) even if mean slightly off target.	E1 E1	2	Large variability unsatisfactory/makes deviation from 135 difficult to detect. Small variability with mean slightly off target may be preferable
Total			17	

MBS4 (cont)

Question Number and Part	Solution	Marks	Total	Comments
4(a)	$\int_0^2 cx + d \, dx = 1$ $\left[\frac{cx^2}{2} + dx \right]_0^2 = 1$ $2c + 2d = 1$	M1 M1 A1	3	Any correct expression - ignore limits Any correct integration - may be awarded elsewhere in question Wholly correct proof allow geometrical proof
(b)(i)	$\int_0^m 0.5 \, dx = 0.5$ $[0.5x]_0^m = 0.5$ $0.5m = 0.5 \quad m = 1$	M1 A1	2	Any correct expression 1 cao by correct method allow geometrical proof
(ii)	$\int_0^m 0.5x \, dx = 0.5$ $\left[\frac{0.5x^2}{2} \right]_0^m = 0.5$ $\frac{0.5m^2}{2} = 0.5$ $m = \sqrt{2} = 1.41$	M1 M1 A1	3	Any correct expression Any correct equation after integration $\sqrt{2}$ or 1.41 (1.41 to 1.42) disallow $\pm\sqrt{2}$
(c)	<p>A not valid - $2c + 2d \neq 1$</p> <p>B valid</p> <p>C not valid - $f(x)$ negative for small x</p>	M1 A1 B1 M1 A1	5	Reason Not valid } Valid } Reason Not valid }
Total			13	

MBS4 (cont)

Question Number and Part	Solution	Marks	Total	Comments																																																		
5(a)	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td></td> <td style="text-align: center;">0</td> <td style="text-align: center;">1</td> <td style="text-align: center;">2</td> <td style="text-align: center;">≥3</td> <td style="text-align: center;">Total</td> </tr> <tr> <td rowspan="2" style="vertical-align: middle; text-align: center;"><2</td> <td style="text-align: center;">1478</td> <td style="text-align: center;">212</td> <td style="text-align: center;">21</td> <td style="text-align: center;">6</td> <td rowspan="2" style="vertical-align: middle; text-align: center;">1717</td> </tr> <tr> <td></td> <td></td> <td colspan="2" style="text-align: center;">27</td> </tr> <tr> <td></td> <td style="text-align: center;">1508.94</td> <td style="text-align: center;">191.24</td> <td style="text-align: center;">14.36</td> <td style="text-align: center;">2.45</td> <td rowspan="3" style="vertical-align: middle; text-align: center;">3185</td> </tr> <tr> <td></td> <td></td> <td></td> <td colspan="2" style="text-align: center;">16.81</td> </tr> <tr> <td></td> <td style="text-align: center;">2830</td> <td style="text-align: center;">334</td> <td style="text-align: center;">20</td> <td style="text-align: center;">1</td> </tr> <tr> <td></td> <td style="text-align: center;">2799.06</td> <td style="text-align: center;">374.76</td> <td style="text-align: center;">26.64</td> <td style="text-align: center;">4.55</td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td colspan="2" style="text-align: center;">31.19</td> <td></td> </tr> <tr> <td style="text-align: center;">Total</td> <td style="text-align: center;">4308</td> <td style="text-align: center;">546</td> <td style="text-align: center;">41</td> <td style="text-align: center;">7</td> <td style="text-align: center;">4902</td> </tr> </table> <p>H₀: No association between number of claims and time for which policy held. H₁: Number of claims associated with time for which policy held.</p> $\sum \frac{(O - E)^2}{E} = \frac{30.94^2}{1508.94} + \frac{30.94^2}{2799.06} + \frac{20.76^2}{191.24} + \frac{20.76^2}{354.76} + \frac{10.19^2}{16.81} + \frac{10.19^2}{31.19}$ <p>= 14.0 c.v. χ^2_2 for 1% risk is 9.21</p> <p>Reject H₀. Conclude number of claims associated with time policy held.</p>		0	1	2	≥3	Total	<2	1478	212	21	6	1717			27			1508.94	191.24	14.36	2.45	3185				16.81			2830	334	20	1		2799.06	374.76	26.64	4.55					31.19			Total	4308	546	41	7	4902	M1 M1 M1 m1		Method for 4 E's Method for all E's Attempt to combine classes Correct method for combining classes - needs all previous M marks
	0	1	2	≥3	Total																																																	
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		B1		Correct H ₀ - may be implied by correctly stated conclusion.																																																		
		M1		Attempt at $\sum \frac{(O - E)^2}{E}$ - allow spurious continuity correction etc																																																		
		m1		Correct use of $\sum \frac{(O - E)^2}{E}$ - requires first two and 4th M mark																																																		
		A1		14.0 (13.9 to 14.1)																																																		
		B1✓		2 df																																																		
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		A1✓	11	Conclusion - mu be compared with upper tail of χ^2 .																																																		
(b)(i), (ii)	<p>A Observed members not frequencies</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td></td> <td style="text-align: center;">Claim</td> <td style="text-align: center;">No claim</td> </tr> <tr> <td style="text-align: center;">Male</td> <td style="text-align: center;">412</td> <td style="text-align: center;">2824</td> </tr> <tr> <td style="text-align: center;">Female</td> <td style="text-align: center;">181</td> <td style="text-align: center;">1485</td> </tr> </table> <p>B Classes overlap</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td></td> <td style="text-align: center;">Claim</td> <td style="text-align: center;">No claim</td> </tr> <tr> <td style="text-align: center;"><25</td> <td style="text-align: center;">280</td> <td style="text-align: center;">676</td> </tr> <tr> <td style="text-align: center;">≥25</td> <td style="text-align: center;">314</td> <td style="text-align: center;">3632</td> </tr> </table> <p>C Classes not complete</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td></td> <td style="text-align: center;">No claim</td> <td style="text-align: center;">≤£2000</td> <td style="text-align: center;">>£2000</td> </tr> <tr> <td style="text-align: center;"><25</td> <td style="text-align: center;">676</td> <td style="text-align: center;">135</td> <td style="text-align: center;">145</td> </tr> <tr> <td style="text-align: center;">≥25</td> <td style="text-align: center;">3632</td> <td style="text-align: center;">122</td> <td style="text-align: center;">192</td> </tr> </table>		Claim	No claim	Male	412	2824	Female	181	1485		Claim	No claim	<25	280	676	≥25	314	3632		No claim	≤£2000	>£2000	<25	676	135	145	≥25	3632	122	192	E1 M1 A1 E1 M1 A1 E1 M1 A1		Reason Method Table cao Reason Method Table cao Reason Method Table cao M1A0 for correct 2×2 table																				
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