

General Certificate of Education
June 2005
Advanced Level Examination



**MATHEMATICS AND STATISTICS
(SPECIFICATION B)
Unit Pure 7**

MBP7

Wednesday 22 June 2005 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 15 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP7.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 Find $\lim_{x \rightarrow 0} \left(\frac{x + \sin x}{1 - e^{-x}} \right)$. (3 marks)

2 On a sketch of the complex plane, shade the region represented by the inequality

$$|z - (2 + i)| < |z + 1|$$
(3 marks)

3 The cubic equation $x^3 - 5x^2 + 6x + 11 = 0$ has roots α , β and γ .

Determine the values of:

(a) $\alpha + \beta + \gamma$; (1 mark)

(b) $\alpha^2 + \beta^2 + \gamma^2$; (3 marks)

(c) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. (2 marks)

4 A line L has cartesian equations $x - 7 = \frac{y + 4}{7} = \frac{z - 37}{6}$.

(a) Find, in the form $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$, a vector equation for L . (2 marks)

(b) Determine the shortest distance from $P(-25, 6, 2)$ to L . (5 marks)

5 The system of equations

$$\begin{aligned}x - y + 2z &= 26 \\2x + y + 3z &= 47 \\4x + 35y - 5z &= 39\end{aligned}$$

is consistent.

(a) Show that this system of equations has no unique solution. (2 marks)

(b) Find the solution of this system of equations. (5 marks)

(c) Interpret this solution geometrically. (1 mark)

- 6 (a) The set S consists of all matrices of the form $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$, where a is a real number.

Show that S , under the operation of matrix multiplication, forms a group G .

(You may assume that matrix multiplication is associative.) (4 marks)

- (b) Determine, giving a reason in each case, whether the group G is isomorphic to:

(i) the set of real numbers under addition; (2 marks)

(ii) the set of real numbers under multiplication. (2 marks)

- 7 A curve has polar equation $r = \frac{4}{9} + \ln\left(1 + \frac{\theta}{3}\right)$.

- (a) (i) Write out the series expansion for $\ln\left(1 + \frac{\theta}{3}\right)$ in ascending powers of θ , up to and including the term in θ^2 , and state the range of values of θ for which the full expansion is valid. (3 marks)

- (ii) Use this result to write down a linear approximation for $\frac{dr}{d\theta}$. (1 mark)

- (b) Given that θ is sufficiently small for terms in θ^2 and higher powers of θ to be ignored, show that

$$r^2 + \left(\frac{dr}{d\theta}\right)^2 \approx \frac{1}{81}(25 + 18\theta) \quad (3 \text{ marks})$$

- (c) Hence find an approximation to the length of the arc of this curve between the points where $\theta = -0.5$ and $\theta = 0$, giving your answer to 3 decimal places. (4 marks)

TURN OVER FOR THE NEXT QUESTION

8 An ellipse has parametric form

$$x = 2 \cos \theta, \quad y = \frac{1}{2} \sin \theta \quad (0 \leq \theta \leq 2\pi)$$

- (a) Show that the tangent to this ellipse at the point $P(2 \cos \theta, \frac{1}{2} \sin \theta)$ can be written in the form

$$x \cos \theta + 4y \sin \theta = 2 \quad (4 \text{ marks})$$

- (b) (i) Show that this tangent meets the hyperbola with equation $x^2 - 9y^2 = 9$ when

$$(25 \sin^2 \theta - 9)y^2 - (16 \sin \theta)y + (9 \sin^2 \theta - 5) = 0 \quad (4 \text{ marks})$$

- (ii) The tangent to the ellipse is also a tangent to the hyperbola.

Find all possible values of $\sin \theta$. (6 marks)

END OF QUESTIONS