

General Certificate of Education
June 2004
Advanced Level Examination



**MATHEMATICS AND STATISTICS
(SPECIFICATION B)
Unit Pure 7**

MBP7

Wednesday 23 June 2004 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 15 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP7.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 (a) For $0 \leq x \leq 2\pi$, draw the graph of $y = 1 + \cos 3x$. (2 marks)
- (b) Sketch the curve with polar equation $r = 1 + \cos 3\theta$ ($0 \leq \theta \leq 2\pi$). (4 marks)

- 2 (a) Show that $(a - c)$ is a factor of

$$\Delta = \begin{vmatrix} 1 & -2 & 1 \\ a & b & c \\ b+c & c+a & a+b \end{vmatrix}$$

and factorise Δ completely. (5 marks)

- (b) Hence, or otherwise, show that the three planes

$$\begin{aligned} x - 2y + z &= 10 \\ 5x + 7y + 5z &= 21 \\ 12x + 10y + 12z &= 35 \end{aligned}$$

do not meet at a single point. (2 marks)

- 3 (a) Show that the series expansion of $e^x + \sin x$ is

$$1 + 2x + \frac{1}{2}x^2 + px^3 + qx^4 + \dots$$

stating the value of p and the value of q . (2 marks)

- (b) Given that

$$(1 + ax)^n = 1 + 2x + \frac{1}{2}x^2 + kx^3 + \dots$$

where a , n and k are constants:

- (i) find the value of a and the value of n ; (4 marks)
- (ii) determine the value of k ; (1 mark)
- (iii) state the range of values of x for which the expansion of $(1 + ax)^n$ is valid. (1 mark)

4 A rectangular hyperbola is represented parametrically by

$$x = 2t, \quad y = -\frac{2}{t}, \quad t \neq 0$$

(a) Show that the normal to this hyperbola has equation

$$y + t^2x = \frac{2}{t}(t^4 - 1) \quad (4 \text{ marks})$$

(b) The normal meets the x -axis at P and the y -axis at Q . The midpoint of PQ is M . Determine a cartesian equation for the locus of M as t varies. You need not simplify your answer. (5 marks)

5 The group G consists of the set of functions f, f^2, f^3, \dots , under the operation of composition of functions, defined for complex numbers z by

$$f(z) = f^1(z) = iz + i$$

and $f^n(z) = f\{f^{n-1}(z)\}$ for $n \geq 2$

(a) (i) Show that $f^4(z) = z$. (4 marks)

(ii) Hence describe the group G as fully as possible. (2 marks)

(b) State Lagrange's Theorem for groups of finite order, and use it to find possible orders for subgroups of G . For each possible order, describe the corresponding subgroup. (3 marks)

6 (a) A line has vector equation $\mathbf{r} = \mathbf{a} + \lambda \mathbf{d}$.

(i) Explain the significance of the vectors \mathbf{a} and \mathbf{d} in relation to the line. (1 mark)

(ii) By considering a suitable vector product, show that the equation of the line can also be expressed in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{d} = \mathbf{0}$. (2 marks)

(b) Two lines have equations

$$(\mathbf{r} - (3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})) \times (2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}) = \mathbf{0}$$

$$\text{and } (\mathbf{r} - (\mathbf{i} + 2\mathbf{j} + \mathbf{k})) \times (\mathbf{i} - 3\mathbf{j} + \mathbf{k}) = \mathbf{0}$$

(i) Determine $(2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}) \times (\mathbf{i} - 3\mathbf{j} + \mathbf{k})$. (2 marks)

(ii) Find, in surd form, the shortest distance between these lines. (3 marks)

- 7 (a) In the complex plane, the circle C is described by the equation

$$|z - 2 + i| = \sqrt{3}$$

- (i) State the complex number which is the centre of C . *(1 mark)*
- (ii) Sketch C on an Argand diagram. *(1 mark)*
- (iii) Write down a cartesian equation for C . *(1 mark)*
- (b) The half-line H has cartesian equation

$$y = mx - 1, \quad x > 0$$

You are given that $m > 0$ and that H is a tangent to C .

- (i) Sketch H on your Argand diagram. *(1 mark)*
- (ii) Show that $m = \sqrt{3}$. *(4 marks)*
- (iii) Express the locus of H in the form $\arg(z - \alpha) = \theta$. *(2 marks)*
- (iv) Determine the complex number which is represented by the point of contact of H and C . *(3 marks)*

END OF QUESTIONS