General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) **MBP6**
Unit Pure 6

Tuesday 28 June 2005 Afternoon Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP6.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 Show that the radius of curvature, at the point where $x = a$, of the curve with equation $y = \cosh x$ is $\cosh^2 a$. *(4 marks)*

2 By writing $\sinh y = x$, prove that

$$\sinh^{-1} x = \ln\left\{x + \sqrt{x^2 + 1}\right\}$$ *(4 marks)*

3 (a) Solve the equation $7 - \cosh x = \sinh x$, giving your answer in the form $\ln k$, where k is an integer. *(3 marks)*

(b) On the same diagram, sketch the graphs of $y = \sinh x$ and $y = 7 - \cosh x$, indicating the coordinates of the points where the curves cross the axes. *(4 marks)*

4 A curve is defined parametrically by

$$x = t^2 + \tfrac{2}{3}t^3, \quad y = t + t^2$$

(a) Verify that $\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = (1 + 2t + 2t^2)^2$. *(3 marks)*

(b) (i) Determine the length of the arc of this curve between the points where $t = 0$ and $t = 1$. *(3 marks)*

(ii) When this arc is rotated about the x-axis through 2π radians, it generates a surface of revolution with area S. Find S in terms of π. *(4 marks)*

5 (a) (i) Show that $\dfrac{1}{\cos\theta + \mathrm{i}\sin\theta} = \cos\theta - \mathrm{i}\sin\theta$. *(1 mark)*

(ii) Given that $z = \cos\dfrac{\pi}{4} + \mathrm{i}\sin\dfrac{\pi}{4}$, evaluate

$$\left(z + \frac{1}{z}\right)^{12}$$ *(4 marks)*

(b) (i) Use de Moivre's Theorem to simplify the expression

$$[r(\cos\theta + \mathrm{i}\sin\theta)]^{-n}$$ *(1 mark)*

(ii) Using this result, or otherwise, find the integer m for which

$$\left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\right)^{-12} = m$$ *(5 marks)*

6 All points (x, y) on a particular curve satisfy the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 4\frac{\mathrm{d}y}{\mathrm{d}x} + 4y = 4 + 27\mathrm{e}^{-x}$$

The point $P(0, 6)$ lies on this curve, and the gradient of the curve at P is zero.

(a) Without solving the differential equation, evaluate $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at P and hence describe P as fully as possible. *(3 marks)*

(b) By solving the differential equation, find the equation of this curve. *(11 marks)*

7 (a) Use the identity $\sin(A+B) + \sin(A-B) = 2\sin A\cos B$ to prove that:

(i) $\sin 2\theta = 2\sin\theta\cos\theta$; *(1 mark)*

(ii) $\sin 3\theta + \sin\theta = 4\sin\theta - 4\sin^3\theta$. *(4 marks)*

(b) Show that $(2s - 1)$ is a factor of $8s^3 - 8s + 3$. *(1 mark)*

(c) Solve the equation

$$\sin 3\theta + \sin\theta = \tfrac{3}{2}$$

giving your answers in radians in the interval $0 < \theta < \pi$. *(9 marks)*

8 (a) (i) Write down the matrix $\boldsymbol{R}$ which represents an anticlockwise rotation through $\frac{\pi}{4}$ radians about O. *(1 mark)*

(ii) Describe the transformation with matrix $\boldsymbol{S} = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$, where k is a positive constant. *(2 marks)*

(iii) Show that $\boldsymbol{R}^{-1}\boldsymbol{SR} = \frac{1}{2}\begin{bmatrix} k+2 & k \\ -k & 2-k \end{bmatrix}$. *(3 marks)*

(b) Show that the matrix $\boldsymbol{M} = \begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix}$ has just one eigenvalue λ. State the value of λ and find a corresponding eigenvector. *(4 marks)*

(c) The transformation **T** is defined by $\begin{bmatrix} x' \\ y' \end{bmatrix} = \boldsymbol{M}\begin{bmatrix} x \\ y \end{bmatrix}$.

(i) Write down a cartesian equation of the line of invariant points of **T**, and give a reason for your choice of answer. *(2 marks)*

(ii) Show that, for some value of k, $\boldsymbol{M} = \boldsymbol{R}^{-1}\boldsymbol{SR}$ and hence give a full geometric description of **T**. *(3 marks)*

END OF QUESTIONS