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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics and Statistics 6320
Specification B

MBP6 Pure 6

Mark Scheme

2005 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
–*x* ee		deduct *x* marks for each error
pi		possibly implied
sca		substantially correct approach

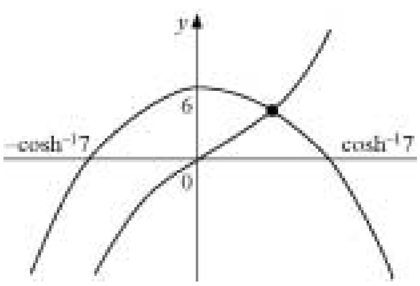
Abbreviations used in Marking

MC – *x*	deducted *x* marks for mis-copy
MR – *x*	deducted *x* marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

Application of Mark Scheme

No method shown:	
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

Mathematics and Statistics B Pure 6 MBP6 June 2005

Q	Solutions	Marks	Total	Comments
1	$\frac{dy}{dx} = \sinh x$ and $\frac{d^2y}{dx^2} = \cosh x$	B1		Both
	Attempt at $\rho = \left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2} \div \frac{d^2y}{dx^2}$	M1		
	Use of $1 + \text{sh}^2 = \text{ch}^2$	B1		
	$\rho = \cosh^2 a$	A1	4	**ag**
	Total		**4**	
2	$x = \sinh y = \frac{1}{2}(e^y - e^{-y}) \Rightarrow 2x\,e^y = e^{2y} - 1$	M1		Attempt to get a quadratic in e^y
	$\Rightarrow (e^y)^2 - 2x(e^y) - 1 = 0$	A1		
	$\Rightarrow e^y = \frac{2x \pm \sqrt{4x^2+4}}{2}$	M1		
	$\Rightarrow y = \ln\left\{x + \sqrt{x^2+1}\right\}$	A1	4	MUST include explanation of choice of sign; e.g. $e^y > 0$
	Total		**4**	
3(a)	$7 - \frac{1}{2}(e^x + e^{-x}) = \frac{1}{2}(e^x - e^{-x})$	M1		
	$\Rightarrow e^x = 7$	A1		cao
	$\Rightarrow x = \ln 7$	A1√	3	
(b)		B1		$y = 7 - \cosh x$ generally OK
		B1		$y = \sinh x$ OK
		B1		For exactly one pt. of intersection
		B1	4	For $(0, 6)$ and $(\pm \cosh^{-1} 7, 0)$ N.B. or ± 2.63 or $\pm \ln(7 \pm \sqrt{48})$
			7	

MBP6 (cont)

Q	Solutions	Marks	Total	Comments
4(a)	$\frac{dx}{dt} = 2t + 2t^2$ and $\frac{dy}{dt} = 1 + 2t$	B1		Both
	$\left(\overset{*}{x}\right)^2 + \left(\overset{*}{y}\right)^2 = (4t^2 + 8t^3 + 4t^4) +$			
	$(1 + 4t + 4t^2)$	M1		
	$= 1 + 4t + 8t^2 + 8t^3 + 4t^4$			
	$= (1 + 2t + 2t^2)$	A1	3	**ag** Verification OK
(b)(i)	$L = \int\left(1+2t+2t^2\right)\, dt = \left[t + t^2 + \frac{2}{3}t^3\right]$	M1 A1		
	$= \frac{8}{3}$	A1	3	
(ii)	$S = 2\pi\int\left(t+t^2\right)\left(1+2t+2t^2\right)\, dt$	M1		
	$= 2\pi\int\left(t+3t^2+4t^3+2t^4\right)\, dt$	A1		
	$= 2\pi\left[\frac{1}{2}t^2 + t^3 + t^4 + \frac{2}{5}t^5\right]$	A1		
	$= 5.8\,\pi$	A1	4	
			10	
5(a)(i)	$(c - is)(c + is) = c^2 - i^2 s^2 = c^2 + s^2 = 1$ $\Rightarrow$ result	B1	1	Or by de Moivre's Theorem etc
(ii)	$z + \frac{1}{z} = 2\cos\frac{\pi}{4} = \sqrt{2}$	M1 A1		
	$\Rightarrow \left(z+\frac{1}{z}\right)^{12} = \left(\sqrt{2}\right)^{12} = 2^6 = 64$	M1 A1	4	
(b)(i)	$\left[r(\cos\theta + i\sin\theta)\right]^{-n} = r^{-n}(\cos n\theta - i\sin n\theta)$	B1	1	
(ii)	$\omega = \frac{1}{2} - \frac{1}{2}\,i = \frac{1}{\sqrt{2}}\left(\cos\frac{\pi}{4} - i\,\sin\frac{\pi}{4}\right)$	B1 B1		Mod Arg oe
	$\Rightarrow$			
	$\omega^{12} = \left(\frac{1}{\sqrt{2}}\right)^{12}\left(\cos\frac{12\pi}{4} + i\,\sin\frac{12\pi}{4}\right)$	M1 A1		
	$= -64$	A1	5	
	Total		**11**	

MBP6 (cont)

Q	Solutions	Marks	Total	Comments
6 (a)	$y'' - 4\times 0 + 4\times 6 = 4 + 27 \Rightarrow y'' = 7$ at P Thus P is a minimum t.p.	M1 A1 B1√	 3	 min/max from their y'' sign
(b)	Aux. Eqn. is $m^2 - 4m + 4 = 0 \;\Rightarrow\; m = 2$ (twice) Giving Comp. Fn. $y_c = (A + Bx)\,e^{2x}$	 M1 A1 B1√		
	For P.I. try $y = \alpha + \beta e^{-x}$ $(y' = -\beta e^{-x},\; y'' = \beta e^{-x})$ $\Rightarrow$ $\beta e^{-x} + 4\beta e^{-x} + 4\alpha + 4\beta e^{-x} = 4 + 27\,e^{-x}$ Comparing terms and solving $\alpha = 1$, $\beta = 3$ or P.I. is $y_p = 1 + 3\,e^{-x}$	 B1 M1 m1 A1		 Substituting into d.e.
	Gen. Soln. is $y = 1 + 3\,e^{-x} + (A + Bx)\,e^{2x}$	B1√		PI (with no arb. consts.) + CF (with two arb. consts.)
	$y' = -3\,e^{-x} + (B + 2A + 2Bx)\,e^{2x}$ Use of $x = 0, y = 6$ and/or $x = 0, y' = 0$ to find A, B $6 = 1 + 3 + A$, $0 = -3 + B + 2A$ $\Rightarrow A = 2,\; B = -1$ i.e. $y = 1 + 3\,e^{-x} + (2 - x)\,e^{2x}$	B1√ M1 A1	 11	GS (with correct # of terms) cao
	Total		**14**	
7 (a)(ii)	$A = B = \theta$ gives $\sin 2\theta (+ \sin 0) = 2 \sin\theta \cos\theta$	 B1	 1	
(ii)	$A = 2\theta, B = \theta \Rightarrow$ $\sin 3\theta + \sin\theta = 2 \sin 2\theta \cos\theta$ Use of (i)'s result: $= 2 \,.\, 2 \sin\theta \cos^2\theta$ Use of $c^2 = 1 - s^2$: $= 4 \sin\theta - 4 \sin^3\theta$	M1 A1 M1 A1	 4	 **ag**
(b)	$f(½) = 0$ shown or $(2s - 1)(4s^2 + 2s - 3)$	B1	1	
(c)	$4s - 4s^3 = \frac{3}{2} \;\Rightarrow\; 8s^3 - 8s + 3 = 0$ $(2s - 1)(4s^2 + 2s - 3) = 0$ $\sin\theta = ½ \;\Rightarrow\; \theta = \dfrac{\pi}{6}$ (or 0.524 rads)	B1 M1A1 M1 A1√		 cao first answer, ft second answer
	$\sin\theta = \dfrac{-2 \pm \sqrt{52}}{8} = 0.6514 \ldots$ $\Rightarrow\; \theta = 0.709$ rads And $\theta = \dfrac{5\pi}{6}$ (or 2.62 rads), 2.43 rads	M1A1 A1√ A1	 9	Ignore $\sin\theta = -1.1514 \ldots$ cao first answer, ft second answer To at least 3 s.f.
			15	

MBP6 (cont)

Q	Solutions	Marks	Total	Comments
8(a)(i)	$\mathbf{R} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$	B1	1	
(ii)	**S** gives a Shear, with x-axis invariant and mapping (e.g.) $(1, 1) \to (2, 1)$	M1 A1	2	Accept "parallel to x-axis, s.f. k". Any pt. (not on x-axis) + image
(iii)	$\mathbf{R}^{-1} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$	B1√		ft if suitable
	$\mathbf{R}^{-1}\,\mathbf{S}\,\mathbf{R} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$	M1		Including good attempt to multiply at least 2 matrices
	$= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1+k & k-1 \\ 1 & 1 \end{bmatrix}$			
	$\left(\text{or } \frac{1}{2}\begin{bmatrix} 1 & k+1 \\ -1 & 1-k \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\right)$			
	$= \frac{1}{2}\begin{bmatrix} k+2 & k \\ -k & 2-k \end{bmatrix}$	A1	3	**ag**
(b)	Char. Eqn. is $\lambda^2 - 2\lambda + 1 = 0 \Rightarrow \lambda = 1$ (twice) Subst^g. $\lambda = 1$ into $\lvert \mathbf{M} - \lambda\mathbf{I} \rvert = 0$ $\Rightarrow 2x + 2y = 0 \Rightarrow$ evec(s). are $\alpha\begin{bmatrix} 1 \\ -1 \end{bmatrix}$	 M1 A1 M1 A1	 4	 Any non-zero α is OK
(c)(i)	$y = -x$ since $\lambda = 1$ gives a line of invariant pts	B1 B1	 2	
(ii)	$k = 4$ gives $\mathbf{M} = \mathbf{R}^{-1}\,\mathbf{S}\,\mathbf{R}$ Hence T is a Shear with $y = -x$ invariant, mapping (e.g.) $(1, 1) \to (5, -3)$	B1 M1 A1	 3	 ft their **S** Fully correct description required
	Total		**15**	
	TOTAL		**80**	