GCE 2004
June Series

Mark Scheme

Mathematics and Statistics B
MBP6

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
–*x* ee		deduct *x* marks for each error
pi		possibly implied
sca		substantially correct approach

Abbreviations used in Marking

MC – *x*	deducted *x* marks for mis-copy
MR – *x*	deducted *x* marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

Application of Mark Scheme

No method shown:	
Correct answer without working	**mark as in scheme**
Incorrect answer without working	**zero marks unless specified otherwise**
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	**mark both/all fully and award the mean mark rounded down**
1 complete and 1 partial attempt, neither crossed out	**award credit for the complete solution only**
Crossed out work	**do not mark unless it has not been replaced**
Alternative solution **using a correct or partially correct method**	**award method and accuracy marks as appropriate**

Mathematics and Statistics B Pure 6 MBP6 June 2004

Question Number and Part	Solution	Marks	Total	Comments
1	$\frac{dy}{dx} = 3\,\text{sech}^2 x \quad - \quad 4\,\text{sech}\,x\tanh x$	B1 B1		
	Setting their $y' = 0$	M1		Or attempt at verification
	Sorting out denominator	m1		Give (as a B1) for $\cosh x = \frac{5}{4}$
	Correctly showing $\sinh x = \frac{3}{4}$	A1	5	Or $y' = 0$ legitimately. **ag**
	Total		**5**	
2 (a)	$r = 256$ and $\theta = 0.8600$	B1 B1	2	r exact; θ to any accuracy
(b)	$\lvert z_1 \rvert = 2$	B1√		ft their $\sqrt[8]{r}$
	$\arg(z_1) = 0.1075$	B1√	2	ft their $\theta \div 8$
(c)	z_1 plotted on an Argand diagram	B1		Must be approx. correct in 1st quad.
	Other seven roots all on circle, centre O and radius 2	B1		Correct distances statement
	and equally spaced (at angles of $\frac{\pi}{4}$) around it	B1	3	Correct angles statement
	Total		**7**	
3 (a)	Aux. eqn. $m^2 + 2m + 1 = 0 \Rightarrow m = -1$ (twice)	M1 A1		
	CF is $y = (Ax + B)\,e^{-x}$	B1√		ft
	For P.I., try $y = a\,e^{3x}$	M1		
	Subst^g. their y, y', y'' into diff. eqn.	m1		
	PI is $y = \frac{1}{2}\,e^{3x}$	A1		i.e. $a = \frac{1}{2}$
	GS is their CF (with 2 arb. Consts.) + their PI (with none): $y = (Ax + B)\,e^{-x} + \frac{1}{2}\,e^{3x}$	B1√	7	ft
(b)	$\frac{dy}{dx} = (A - Ax - B)\,e^{-x} + \frac{3}{2}\,e^{3x}$	B1√		ft valid GS's
	Use of $x = 0$, $y = 1$, $y' = 2$ to find A, B	M1		Either will do
	$A = 1$, $B = \frac{1}{2}$ or $y = (x + \frac{1}{2})\,e^{-x} + \frac{1}{2}\,e^{3x}$	A1	3	cao
	Total		**10**	
4 (a)	$(\sin x + \sin 4x) + (\sin 2x + \sin 3x)$	M1		Or other pairing e.g.$(\sin x + \sin 2x) + (\sin 3x + \sin 4x)$
	$= 2\sin\frac{5}{2}x\cos\frac{3}{2}x + 2\sin\frac{5}{2}x\cos\frac{1}{2}x$	A1 A1		$= 2\sin\frac{3}{2}x\cos\frac{1}{2}x + 2\sin\frac{7}{2}x\cos\frac{1}{2}x$
	Factorisation and repeated use of sum-and-product formulae:	M1		
	$2\sin\frac{5}{2}x\,(\cos\frac{3}{2}x + \cos\frac{1}{2}x)$			$= 2\,(\sin\frac{3}{2}x + 2\sin\frac{7}{2}x)\cos\frac{1}{2}x$
	$= 4\cos\frac{1}{2}x\cos x\sin\frac{5}{2}x$	A1	5	**ag**
(b)	$\cos\frac{1}{2}x = 0$, $\cos x = 0$, $\sin\frac{5}{2}x = 0$	M1		At least one of, incl. solving attempt
	$x = \pi \quad x = \frac{1}{2}\pi \quad x = 0, \frac{2}{5}\pi, \frac{4}{5}\pi$	A1 A1 A1	4	One for each equation's solutions
	Total		**9**	

MBP6 (cont)

Question Number and Part	Solution	Marks	Total	Comments
5 (a)	$(c+\mathrm{i}s)^1 = \cos.1\theta + \mathrm{i}\sin.1\theta$	B1		
	$\Rightarrow$ true for $n = 1$			
	Assuming that $(c+\mathrm{i}s)^k = \cos k\theta + \mathrm{i}\sin k\theta$	B1		Or fully explained later
	$\Rightarrow (c+\mathrm{i}s)^{k+1}$			
	$= (\cos k\theta + \mathrm{i}\sin k\theta)(\cos\theta + \mathrm{i}\sin\theta)$	M1		At least this far
				Legitimately shown via
	$= \cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta$	A1	4	$(C_k C_1 - S_k S_1) + \mathrm{i}(S_k C_1 + C_k S_1)$
(b)(i)	$(-\sqrt{3} + \mathrm{i})^n = 2^n[\cos(\frac{5}{6}n\pi) + \mathrm{i}\sin(\frac{5}{6}n\pi)]$	B1		Dealing with the 2
		M1 A1	3	Dealing with the argument; correct
(ii)	Require $\sin(\frac{5}{6}n\pi) = 0$ **and** $\cos(\frac{5}{6}n\pi) > 0$	M1		
	Least $n = 12$	A1	2	
	Total		**9**	
6 (a)	Char. Eqn. is $\lambda^2 - 25\lambda + 100 = 0$	M1 A1		
	$\Rightarrow \lambda = 5, 20$	B1√		ft provided real
	$\lambda = 5 \Rightarrow 3x + 6y = 0$ or $y = -\frac{1}{2}x \Rightarrow$	M1		Either case attempted
	evecs. $\alpha\begin{bmatrix} 2 \\ -1 \end{bmatrix}$	A1		Any (non-zero) multiple will do
	$\lambda = 20 \Rightarrow -12x + 6y = 0$ or $y = 2x \Rightarrow$			
	evecs. $\beta\begin{bmatrix} 1 \\ 2 \end{bmatrix}$	A1	6	
(b) (i)	Invariant lines have gradients $-\frac{1}{2}$ and 2	B1		
	Product of gradients = – 1 $\Rightarrow$ lines perpendicular	B1	2	**Or** M1 A1 via scalar prod. = 0
(ii)	Two-way stretch	M1		Or the composition of 2 stretches
	Parallel to $y = -\frac{1}{2}x$ of s.f. 5	A1		
	and parallel to $y = 2x$ of s.f. 20	A1	3	
	Total		**11**	

MBP6 (cont)

Question Number and Part	Solution	Marks	Total	Comments
7 (a)	$\dfrac{t}{\sqrt{1+t^2}}$	B1	1	
(b) (i)	$I_n = \displaystyle\int t^{n-1} \frac{t}{\sqrt{1+t^2}}\, dt$	M1		Splitting of terms + parts attempt
	$= t^{n-1}\sqrt{1+t^2} - \displaystyle\int \sqrt{1+t^2}\,(n-1)\,t^{n-2}\, dt$	A1 A1		
	$= \sqrt{2} - (n-1)\displaystyle\int \frac{(1+t^2)\,t^{n-1}}{\sqrt{1+t^2}}\, dt$	M1		
	$\Rightarrow I_n = \sqrt{2} - (n-1)(I_{n-2} + I_n)$			
	$\Rightarrow n\,I_n = \sqrt{2} - (n-1)\,I_{n-2}$	A1	5	**ag**
(ii)	$I_1 = \sqrt{2} - 1$	B1		
	Use of redn. formula for case $n = 3$	M1		$I_3 = \frac{1}{3}\{\sqrt{2} - 2\,I_1\}$
	$I_3 = \frac{1}{3}\{2 - \sqrt{2}\}$	A1	3	Any correct surd form
(c)	$t = \tan\frac{1}{2}x \Rightarrow dx = \dfrac{2}{1+t^2}\,dt$	B1		Or equivalent work
	Full substn. to eliminate x	M1		
	$I = 2I_3 = \frac{2}{3}\{2 - \sqrt{2}\}$	A1$\surd$	3	ft suitable I_3 s
	Total		**12**	

MBP6 (cont)

Question Number and Part	Solution	Marks	Total	Comments
8 (a) (i)	$2\sinh\theta\cosh\theta$			
	$= 2\times\frac{1}{2}(e^{\theta}-e^{-\theta})\times\frac{1}{2}(e^{\theta}+e^{-\theta})$	M1		
	$= \frac{1}{2}(e^{2\theta}-e^{-2\theta}) = \sinh 2\theta$	A1		**ag**
(ii)	$2\sinh^2\theta = 2\times\frac{1}{4}(e^{\theta}-e^{-\theta})^2$	M1		
	$= \frac{1}{2}(e^{2\theta}+e^{-2\theta}) - 1 = \cosh 2\theta - 1$	A1	4	**ag**
(b) (i)	$\cosh\theta = 2x+1 \Rightarrow \sinh\theta\, d\theta = 2\, dx$	B1		
	and $\sqrt{4x^2+4x} = \sinh\theta$	B1		
	Then $I = \int \sinh\theta \,.\, \frac{1}{2}\sinh\theta\, d\theta$	M1 A1	4	i.e. $k = \frac{1}{2}$
(ii)	$= \frac{1}{4}\int(\cosh 2\theta - 1)\, d\theta$	M1		
	$= \frac{1}{4}\left[\frac{1}{2}\sinh 2\theta - \theta\right]$	A1		
	$= \frac{1}{4}\sinh\theta\cosh\theta - \frac{1}{4}\theta + C$	M1		
	$= \frac{1}{4}\sqrt{4x^2+4x}\,.\,(2x+1)$ $- \frac{1}{4}\cosh^{-1}(2x+1) + C$	A1	4	**ag**
(c)	$L = \int\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = \int\sqrt{1+4x+4x^2}\, dx$	M1 A1		
	$= \int(2x+1)\, dx$	B1		
	$= \left[x^2 - x\right]_{77}^{89}$	A1√		ft integration (linear only)
	$= 2004$	A1	5	cao
	Total		**17**	
	TOTAL		**80**	