GCE 2005
January Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics and Statistics B
(MBP6)

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Key to Mark Scheme

M .... mark is for	method
m .... mark is dependent on one or more M marks and is for	method
A .... mark is dependent on M or m marks and is for	accuracy
B .... mark is independent of M or m marks and is for	method and accuracy
E .... mark is for	explanation
√ or ft or F	follow through from previous incorrect result
CAO	correct answer only
AWFW	anything which falls within
AWRT	anything which rounds to
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
SF	significant figure(s)
DP	decimal place(s)

Abbreviations used in Marking

MC – *x*	deducted *x* marks for mis-copy
MR – *x*	deducted *x* marks for mis-read
ISW	ignored subsequent working
BOD	given benefit of doubt
WR	work replaced by candidate
FB	formulae booklet

Application of Mark Scheme

No method shown:

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

More than one method/choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

Mathematics and Statistics B Pure 6 MBP6 January 2005

Question Number and Part	Solution	Marks	Total	Comments
1	Aux. eqn. is $4m^2 - 8m + 5 = 0$	B1		
	Solving: $m = 1 \pm \frac{1}{2}\text{i}$	M1 A1		
	G.S. is $y = \text{e}^x(A\cos\frac{1}{2}x + B\sin\frac{1}{2}x)$	B1√ B1√	5	Give one B1 only for real roots followed through correctly
	Total		**5**	
2(a)	$\int (\cosh x + \text{sech}^2 x)\,\text{d}x$	M1		Ignore limits until final answer
	$= \sinh x + \tanh x$	A1 A1		
	$= 1.35$	A1	4	
	Total		**4**	
3(a)(i)	$\lvert \pm(u - v) \rvert$	B1		
(a)(ii)	$\arg u - \arg v$	B1	2	
(b)	Clearly indicated parallelogram with W at end of main diagonal **or** Vector triangle with sides **u** , **v** , **w**	B1	1	
	Total		**3**	
4(a)	$\frac{\text{d}x}{\text{d}t} = \frac{2}{t}$ and $\frac{\text{d}y}{\text{d}t} = 1 - \frac{1}{t^2}$	B1 B1		
	$\left(\frac{\text{d}x}{\text{d}t}\right)^2 + \left(\frac{\text{d}y}{\text{d}t}\right)^2 = \frac{4}{t^2} + 1 - \frac{2}{t^2} + \frac{1}{t^4}$	M1		
	$= \left(\frac{t^2+1}{t^2}\right)^2$	A1	4	Legitimately shown
(b)	$S = 2\pi \int \left(\frac{t^2+1}{t}\right)\left(\frac{t^2+1}{t^2}\right)\text{d}t$	B1		
	$= 2\pi \int \left(\frac{t^4+2t^2+1}{t^3}\right)\text{d}t$	M1		Helpful simplification
	$= 2\pi \int \left(t + \frac{2}{t} + \frac{1}{t^3}\right)\text{d}t$	A1		Suitable form for integrating
	$= 2\pi\left[\frac{1}{2}t^2 + 2\ln t - \frac{1}{2t^2}\right]$	A1√ A1√		for the log term for the other (two) terms
	$= \pi\left[\frac{15}{4} + 4\ln 2\right]$	A1	6	cao (any correct exact form)
	Total		**10**	

MBP6 (cont)

Question Number and Part	Solution	Marks	Total	Comments
5(a)	$LHS \equiv 2\left(\frac{1}{2}\left[e^x - e^{-x}\right]\right)\left(\frac{1}{2}\left[e^x + e^{-x}\right]\right)$	M1		
	$\equiv \frac{1}{2}\left[e^{2x} - e^{-2x}\right] \equiv \sinh 2x \equiv RHS$	A1	2	
(b)	I.F. is $\exp\left\{\int \tanh x \ \ dx\right\}$	B1		
	$= \exp\left\{\ln(\cosh x)\right\} = \cosh x$	M1 A1		
	Then d.e. becomes			
	$\frac{d}{dx}\left(y\cosh x\right) = \frac{1}{2}\sinh 2x$	B1		RHS in integrable form
	$\int RHS = \frac{1}{4}\cosh 2x$ or $\frac{1}{2}\cosh^2 x$ etc.	A1√		
	Use of $x = 0$, $y = 1$ to find const. of $\int$	M1		
	$y\cosh x = \frac{3}{4} + \frac{1}{4}\cosh 2x$	A1	7	Including fully correct solution A0 for correct C found from incorrect division by cosh x.
	Total		**9**	

MBP6 (cont)

Question Number and Part	Solution	Marks	Total	Comments
6(a)	$\frac{2t}{1+t^2}.t+\frac{2(1+t^2)}{1-t^2}$	M1		Use of correct half-angle forms for sin x and cos x
	$=\frac{2t^2(1-t^2)+2(1+t^2)^2}{(1+t^2)(1-t^2)}$			
	$=\frac{2+6t^2}{(1+t^2)(1-t^2)}$	A1	2	Answer given
(b)	$4+12t^2+5-5t^4=0$	M1		Polynomial attempt
	$5t^4-12t^2-9=0$	A1		
	(Since $t^2>0$) $t^2=3$	B1√		ft (the) positive root for t^2
	$\tan\frac{1}{2}x=\pm\sqrt{3}$	M1		Including attempt to solve for x
	$x=\frac{2\pi}{3},\ \frac{4\pi}{3}$ (decimals, in radians, OK)	A1√ A1	6	ft first answer For both A's, two correct answers + no extras
(c)	**(i)** $\int\frac{3(1-t^2)(1+t^2)}{2+6t^2}.\frac{2\ \mathrm{d}t}{1+t^2}$	M1 B1		Complete substn. method dx in terms of t's
	$=\int\frac{3-3t^2}{1+3t^2}\mathrm{d}t$	A1		
	Upper limit $=\frac{1}{\sqrt{3}}$	B1	4	
	(ii) $=\int\left(\frac{4}{1+3t^2}-1\right)\mathrm{d}t$	B1		Separated into integrable bits
	$=-t+\frac{4}{\sqrt{3}}\tan^{-1}(t\sqrt{3})$	M1 A1		Must be arctan for the M
	$=\frac{\pi-1}{\sqrt{3}}$	A1	4	cao
	Total		**16**	

MBP6 (cont)

Question Number and Part	Solution	Marks	Total	Comments
7(a)	$(1 + \text{i} \tan \theta)^3$ expanded	M1		Multn. or binomial theorem
	Re. part = $1 - 3 \tan^2\theta$	A1	2	Ignore Im. parts
(b)	$(1 + \text{i} \tan \theta)^3 = \left(\dfrac{\cos\theta + \text{i} \sin\theta}{\cos\theta}\right)^3$	B1		
	$= \left(\dfrac{\cos 3\theta + \text{i} \sin 3\theta}{\cos^3\theta}\right)$	M1		Use of de Moivre's theorem
	Equating Re. parts $\Rightarrow$			
	$1 - 3 \tan^2\theta = \dfrac{\cos 3\theta}{\cos^3\theta}$	A1	3	
	Total		**5**	
8(a)	Char. Eqn. is $\lambda^2 - 9\lambda + 8 = 0$	M1 A1		
	$\lambda = 1$ or 8	A1		
	$\lambda = 1 \Rightarrow 2x + 5y = 0$	M1		Either eval. substd. back
	gives eigenvectors $p\begin{pmatrix} 5 \\ -2 \end{pmatrix}$	A1		
	$\lambda = 8 \Rightarrow -5x + 5y = 0$			
	gives eigenvectors $q\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	A1	6	Any non-zero p , q will serve
(b)(i)	$x = 2,\ y = -1$ substd. in x' & y'	M1		Both x' *and* y' eqns.
	to get $x' = 2,\ y' = -1$	A1	2	
(ii)	$\begin{pmatrix} x'-2 \\ y'+1 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 2 & 6 \end{pmatrix}\begin{pmatrix} x-2 \\ y+1 \end{pmatrix}$	B1	1	i.e. $\alpha = 2, \beta = -1$
(iii)	$2(x - 2) + 5(y + 1) = 0$			
	Or $2x + 5y + 1 = 0$ or equivalent	B2,1√	2	Give B1 for $2x + 5y = 0$ or ft from their eval. of 1
(c)	E.g. $x' = 3x + 5(x + c) + 1 = 8x + 5c + 1$	B1		Use of $y = x + c$ at any stage
	$y' = 2x + 6(x + c) + 1 = 8x + 6c + 1$	M1		
	$= x' + c$	A1	3	
	Total		**14**	

MBP6 (cont)

Question Number and Part	Solution	Marks	Total	Comments
9(a)	$I_n = \left[\frac{1}{3}e^{3x}\tan^n x\right] -$	M1 A1		
	$\int \frac{1}{3}e^{3x} n.\tan^{n-1} x.\sec^2 x\,dx$	A1		
	$\Rightarrow 3\,I_n =$ $\left(\sqrt{3}\right)^n.e^{\pi} - n\int e^{3x}\tan^{n-1} x(1+\tan^2 x)\,dx$	M1		Use of $\sec^2 = 1 + \tan^2$ to get I_{n-1} and I_{n+1} involved
	$= \left(\sqrt{3}\right)^n.e^{\pi} - n\{I_{n-1} + I_{n+1}\}$			
	$\Rightarrow\; n\,I_{n+1} + 3\,I_n + n\,I_{n-1} = \left(\sqrt{3}\right)^n.e^{\pi}$	A1	5	Answer given
	<u>ALTERNATIVE</u>:			
	$I_n = \int e^{3x}\tan^{n-2} x(\sec^2 x - 1)\,dx$	(M1)		
	$= \left[e^{3x}\frac{\tan^{n-1} x}{n-1}\right] - \int\frac{\tan^{n-1} x}{n-1}.3e^{3x}\,dx - I_{n-2}$	(A1) (A1)		
	$\Rightarrow (n-1)(I_n + I_{n-2}) = \left(\sqrt{3}\right)^{n-1}.e^{\pi} - 3\,I_{n-1}$	(M1)		
	$\Rightarrow$ result (one step down)	(A1)		
(b)(i)	$n = 1 \;\Rightarrow\; I_2 + 3\,I_1 + I_0 = \left(\sqrt{3}\right)e^{\pi}$	B1		
	$n = 3 \;\Rightarrow\; 3\,I_4 + 3\,I_3 + 3\,I_2 = \left(\sqrt{3}\right)^3 e^{\pi}$	B1		
	Thus $I_4 + I_3 + I_2 = \sqrt{3}\,e^{\pi} = I_2 + 3\,I_1 + I_0$	M1 A1		
	$\Rightarrow\; I_4 + I_3 - 3\,I_1 = I_0$	A1	5	Answer given
(b)(ii)	Use of $\sec^2 x = 1 + \tan^2 x$ to get	M1		
	$I = I_4 + I_3 - 3\,I_1$	A1		
	$I_0 = \int e^{3x}dx$	M1		Attempt to integrate this
	$= \frac{1}{3}\left(e^{\pi} - 1\right)$	A1	4	
	Total		**14**	
	TOTAL		**80**	