ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark scheme January 2004

GCE

Mathematics & Statistics B

Unit MBP6

Key to mark scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m mark and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
– x EE		Deduct x marks for each error
NMS		No method shown
PI		Perhaps implied
c		Candidate

Abbreviations used in marking

MC – x	deducted x marks for miscopy
MR – x	deducted x marks for misread
ISW	ignored subsequent working
BOD	gave benefit of doubt
WR	work replaced by candidate

Application of mark scheme

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

Question Number and part	Solution	Marks	Total	Comments
1	Attempt to integrate $\frac{1}{x(x-1)} = -\frac{1}{x} + \frac{1}{x-1}$	M1A1		
	$\int = -\ln x + \ln(x-1)$	A1√		ft
	I.F. is $\exp\{\text{this}\} = \frac{x-1}{x}$	M1A1	5	Allow verification: mult[g]. by given I.F. and showing $\text{LHS} = \frac{d}{dx}\left(\frac{y(x-1)}{x}\right)$
	ALTERNATIVE:			
	$\frac{1}{x(x-1)} = \frac{1}{\left(x-\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$	(M1) (A1)		
	So $\int = \frac{1}{2\times\frac{1}{2}} \ln\left\lvert\frac{x-\frac{1}{2}-\frac{1}{2}}{x-\frac{1}{2}+\frac{1}{2}}\right\rvert$	(A1)		From Formula Book
	I.F. is $\exp\{\text{this}\} = \frac{x-1}{x}$	(M1) (A1)	5	
	Total		**5**	
2(a)	$2\sin 4x \cos 3x = \sin 7x + \sin x$	M1A1	2	
(b)	Use of $\int(\sin 7x + \sin x)\,dx$	M1√		ft (a) + integration attempt
	$I = \frac{1}{2}\left[-\frac{1}{7}\cos 7x - \cos x\right]$	A1A1		Ignore the factor $\frac{1}{2}$ until end A1 A0 if both positive
	$= \frac{1}{2}\left[-\frac{1}{7}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} + \frac{1}{7} + 1\right]$	M1		Substitution of limits with exact values attempted;
	$= \frac{2}{7}\left[2-\sqrt{2}\right]$	A1	5	cao, any exact equivalent form
	Total		**7**	
3(a)	Attempt to solve aux. eqn. $m^2 - 5m = 0$	M1		
	$\Rightarrow m = 0, 5$	A1		
	GS is $y = A + Be^{5x}$	B1√	3	ft
(b)	$\frac{dy}{dx} = 2ax + b$ and $\frac{d^2y}{dx^2} = 2a$	B1		
	Substituting these into $y'' - 5y' = 20x$	M1		$2a - 5(2ax + b) = 20x$
	Solving $-10a = 20$ and $2a - 5b = 0$	M1√		ft sim. eqns. from equating terms
	$a = -2,\ b = -\frac{4}{5}$	A1	4	
(c)	GS is $y = A + Be^{5x} - 2x^2 - \frac{4}{5}x$	B1√	1	ft (a) and (b)
	Total		**8**	

Question Number and part	Solution	Marks	Total	Comments
4(a)	$y = \sinh^2 x \Rightarrow \frac{dy}{dx} = 2\sinh x\cosh x$ $= \sinh 2x$	B1	1	
(b)	$\frac{d^2y}{dx^2} = 2\cosh 2x$	B1		oe
	$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \sinh^2 2x = \cosh^2 2x$	M1A1		
	Use of $\kappa = \frac{y''}{\left(1+(y')^2\right)^{\frac{3}{2}}} = \frac{2\cosh 2x}{\cosh^3 2x}$	M1		Or $\rho = \frac{1}{\kappa}$
	$= \frac{2}{\cosh^2 2x}$	A1		
	$= \frac{2}{\frac{1}{2}+\frac{1}{2}\cosh 4x} = \frac{4}{1+\cosh 4x}$	M1A1	7	**ag**
	Total		**8**	
5(a)	Char. Eqn. is $\lambda^2 - 7\lambda - 8 = 0$	M1A1		
	$\Rightarrow \lambda = -1, 8$	A1√		ft if suitable
	$\lambda = -1 \Rightarrow 2x + y = 0$ or $y = -2x \Rightarrow$	M1		Either case attempted
	evecs. $\alpha\begin{bmatrix}1\\-2\end{bmatrix}$	A1		Any (non-zero) multiple will do
	$\lambda = 8 \Rightarrow -5x + 2y = 0$ or $y = \frac{5}{2}x \Rightarrow$			
	evecs. $\beta\begin{bmatrix}2\\5\end{bmatrix}$	A1	6	
(b)(i)	(0 , 0)	B1	1	Accept "The origin" or " *O* "
(ii)	$y = -2x$ and $y = \frac{5}{2}x$	B1√		ft (a)
	$\lambda \neq 1$ in either case	E1	2	oe
	Total		**9**	

Question Number and part	Solution	Marks	Total	Comments
6(a)	mod(8i) = 8 and arg(8i) = $\frac{\pi}{2}$	B1B1	2	
(b)	$z^3 = \left(8, \frac{\pi}{2}\right), \left(8, \frac{5\pi}{2}\right), \left(8, -\frac{3\pi}{2}\right)$	B1		
	$\Rightarrow z = \left(2, \frac{\pi}{6}\right), \left(2, \frac{5\pi}{6}\right), \left(2, -\frac{\pi}{2}\right)$	B1 M1		Cube root of mods args ÷ 3
	$= 2e^{\frac{\pi i}{6}}, \quad 2e^{\frac{5\pi i}{6}}, \quad 2e^{-\frac{\pi i}{2}}$	A1	4	All 3 correct, any polar form (allow final answer with $\frac{3\pi}{2}$)
(c)	Argand diagram: All points equidistant from O Equally spaced around circle	 B1 B1	 2	 All on circle, centre O, radius 2 At 30°, 150°, 270°
(d)	Euler's Rule or from diagram: $2(\cos\theta + i\sin\theta)$ $\sqrt{3} + i, \quad -\sqrt{3} + i, \quad -2i$	 M1√ A1A1	 3	 Any one case ft Any one correct; all 3 correct
	Total		**11**	

Question Number and part	Solution	Marks	Total	Comments
7(a)(i)	$\sec x + \tan x \equiv \frac{1+t^2}{1-t^2} + \frac{2t}{1-t^2}$	B1B1		One for each t-identity used
	$\equiv \frac{(1+t)^2}{(1-t)(1+t)} \equiv \frac{1+t}{1-t}$	M1A1	4	**ag**
(ii)	$t = \tan \frac{1}{2}x \Rightarrow \frac{dt}{dx} = \frac{1}{2}\sec^2 \frac{1}{2}x$	M1		Allow $x = 2\tan^{-1} t$ and
	$\Rightarrow \frac{dx}{dt} = \frac{2}{1+t^2}$ **ag**	A1	2	$\frac{dx}{dt} = \frac{2}{1+t^2}$ from Formula Book
(b)	$\int \sec x \, dx = \int \frac{1+t^2}{1-t^2} \times \frac{2}{1+t^2} \, dt =$	M1		
	$\int \frac{2}{1-t^2} \, dt$	A1		
	$= \ln \left\| \left(\frac{1+t}{1-t}\right) \right\| + C$	M1 A1		Either from Formula Book or via P.F.s: $\int \left(\frac{1}{1-t} + \frac{1}{1+t}\right) dt = \ln(1-t) + \ln(1+t)$
	$= \ln \| \sec x + \tan x \| + C$	A1	5	**ag**
(c)	$y = \ln(\sec x) \Rightarrow \frac{dy}{dx} = \tan x$	B1		
	and $1 + \left(\frac{dy}{dx}\right)^2 = \sec^2 x$	B1		
	$L = \int \sec x \, dx$	M1A1		
	$= \ln \| \sec x + \tan x \|$	A1		
	$= \ln\left(2+\sqrt{3}\right) - \ln\left(\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)$			
	$= \ln\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right) = \ln(1+r)$ where $r = \left(\frac{2}{\sqrt{3}}\right)$	A1	6	
	Total		**17**	

Question Number and part	Solution	Marks	Total	Comments
8(a)	$10s = 3(1 - s^2) + 5$ from use of $\tanh^2 = 1 - \text{sech}^2$	B1		
	$\Rightarrow 3s^2 + 10s - 8 = 0$	M1A1		Creating a quadratic; correct
	$0 = (3s - 2)(s + 4) \Rightarrow s = \text{sech}\, y = \frac{2}{3}$	M1A1	5	Solving; positive answer only
(b)(i)	$x = \text{sech}\, y = \dfrac{2}{e^y + e^{-y}}$			
	$\Rightarrow x\,e^{2y} - 2\,e^y + x = 0$	M1A1		Quadratic in e^y attempt; correct
	$e^y = \dfrac{2 \pm \sqrt{4 - 4x^2}}{2x} = \dfrac{1}{x}\left(1 \pm \sqrt{1 - x^2}\right)$	M1		
	$y = \ln\left\{\dfrac{1 \pm \sqrt{1 - x^2}}{x}\right\}$	m1		
	$= \ln\left\{\dfrac{1 + \sqrt{1 - x^2}}{x}\right\}$ as $y \geq 0$	A1	5	With correct indication of choice of sign
(ii)	$x = \text{sech}\, y$ and use of implicit diffn.	M1		
	$\Rightarrow -\text{sech}\, y \tanh y \dfrac{dy}{dx} = 1$	A1		
	$\Rightarrow \dfrac{dy}{dx} = -\dfrac{1}{x\sqrt{1 - x^2}}$	A1√		ft sign
	Substituting $x = \dfrac{1}{\sqrt{2}} \Rightarrow \dfrac{dy}{dx} = -2$	M1 A1	5	cao (except ft + 2)
	ALTERNATIVE: Using the Chain Rule to differentiate $y = \ln\left\{\dfrac{1 + \sqrt{1 - x^2}}{x}\right\}$			
	$\dfrac{dy}{dx} = \dfrac{x}{1 + \sqrt{1 - x^2}} \times$	(M1) (A1)		
	$\dfrac{x.\frac{1}{2}\left(1 - x^2\right)^{-\frac{1}{2}} - 2x - \left(1 + \sqrt{1 - x^2}\right)}{1 + \sqrt{1 - x^2}}$	(M1) (A1)		Chain Rule used and diffn. of product or quotient
	Substituting $x = \dfrac{1}{\sqrt{2}} \Rightarrow \dfrac{dy}{dx} = -2$	(A1)	(5)	
	Total		**15**	
	TOTAL		**80**	