

General Certificate of Education
June 2004
Advanced Level Examination



**MATHEMATICS AND STATISTICS
(SPECIFICATION B)
Unit Pure 5**

MBP5

Wednesday 23 June 2004 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 15 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP5.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 (a) Obtain the binomial expansion in ascending powers of x up to and including the term in x^3 of the following, giving each term in its simplest form.

(i) $(1 + x)^{-1}$ *(2 marks)*

(ii) $(1 + 4x)^{\frac{1}{2}}$ *(4 marks)*

- (b) Hence show that, for small values of x ,

$$2(1 + 4x)^{\frac{1}{2}} + \frac{4}{1 + x} \approx 6 + kx^3$$

where k is a constant to be found. *(2 marks)*

- 2 By considering rectangular strips of width 0.5, use the mid-ordinate rule to obtain an approximation for $\int_1^2 \frac{2}{e^{2x} - 1} dx$, giving your answer to 3 decimal places. *(3 marks)*

- 3 (a) Express $6 \cos x - 8 \sin x$ in the form $R \cos(x + \alpha)$, where R is a positive constant and $0^\circ < \alpha < 90^\circ$. Give the value of α to the nearest 0.1° . *(3 marks)*

- (b) Hence find the general solution, in degrees, of the equation

$$6 \cos x - 8 \sin x = 3$$
 (4 marks)

- 4 The gradient of a curve, C , at the point (x, y) is given by

$$\frac{dy}{dx} = \frac{1}{2y(x+2)}, \quad x > 0, \quad y > 0$$

The point $P(1, 1)$ lies on the curve C .

- (a) (i) Write down the gradient of the curve C at the point P . *(1 mark)*

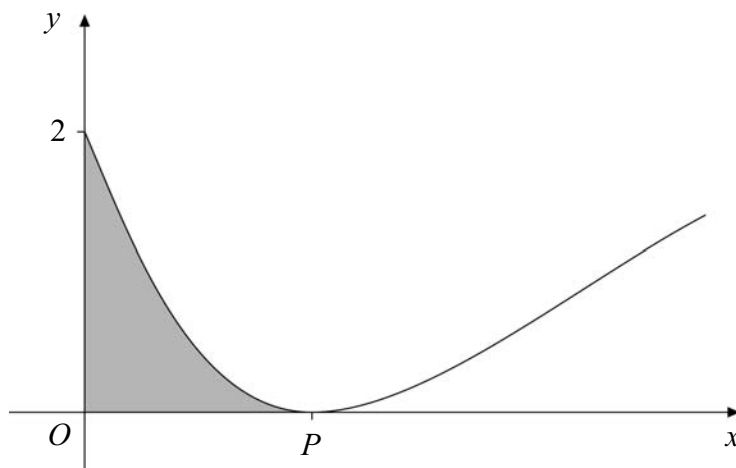
- (ii) Show that the equation of the normal at P is $y + 6x = 7$. *(3 marks)*

- (b) Find the equation of the curve C in the form $y^2 = f(x)$. *(5 marks)*

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- 5 A curve has equation $y = \frac{x^2}{x+1}$.
- (a) Find the equations of the two asymptotes to the curve. *(3 marks)*
- (b) Given that $y \leq -4$ or $y \geq 0$ for all real values of x , and that there are no values of y for which $-4 < y < 0$, find the coordinates of the two turning points of the curve. *(3 marks)*
- (c) Sketch the curve. *(3 marks)*
- 6 (a) Find the cosine of the angle between the vectors $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$. *(3 marks)*
- (b) The equations of two planes are $x - y + 2z = 0$ and $2x + 2y + z = 0$.
- (i) Use your result from part (a) to find the acute angle between the two planes. *(2 marks)*
- (ii) Verify that the point $(-5, 3, 4)$ lies in both planes. *(1 mark)*
- (iii) Given that the origin also lies in both planes, write down a vector equation of the line of intersection of the two planes. *(2 marks)*

TURN OVER FOR THE NEXT QUESTION

7 The diagram shows part of a curve C .



The curve C is defined parametrically by

$$x = t^2, \quad y = 1 + \cos t, \quad 0 \leq t \leq 2\pi$$

The curve C touches the x -axis at the point P .

(a) Show that the x -coordinate of P is π^2 . (2 marks)

(b) Find $\frac{dy}{dx}$ in terms of t . (2 marks)

(c) (i) Show that

$$\frac{d}{dt} \left(\frac{dy}{dx} \right) = \frac{\sin t - t \cos t}{2t^2} \quad (2 \text{ marks})$$

(ii) Hence find $\frac{d^2y}{dx^2}$ in terms of t . (2 marks)

(iii) Hence show that any point of inflection of C must have a parameter t whose value satisfies the equation $\tan t = t$. (1 mark)

(d) (i) Find $\int t \cos t \, dt$. (3 marks)

(ii) Find, in terms of π , the area of the shaded region bounded by the curve C , the y -axis and the line OP . (4 marks)

END OF QUESTIONS