

General Certificate of Education
January 2005
Advanced Level Examination



**MATHEMATICS AND STATISTICS
(SPECIFICATION B)
Unit Pure 5**

MBP5

Thursday 27 January 2005 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 15 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP5.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A curve is given by the equation

$$y = x^2 - e^x$$

(a) Find $\frac{d^2y}{dx^2}$. (2 marks)

(b) Show that $\frac{d^4y}{dx^4} = \frac{d^3y}{dx^3}$ for all values of x . (1 mark)

(c) Show that the curve has only one point of inflection and find its coordinates. (4 marks)

2 Use Simpson's rule with five ordinates (four equal strips) to find an approximation to the integral

$$\int_0^2 \sqrt{1+x^2} \, dx$$

giving your answer to three decimal places. (4 marks)

3 Find the general solution, in radians, of the equation

$$\sin 2x + \cos x = 0 \quad (5 \text{ marks})$$

4 (a) (i) Show that $(2-x)^{-2}$ can be written as $\frac{1}{4} \left(1 - \frac{x}{2}\right)^{-2}$. (1 mark)

(ii) Obtain the first three terms of the binomial expansion of $(2-x)^{-2}$ in ascending powers of x . (3 marks)

(iii) State the range of values of x for which the full expansion is valid. (2 marks)

(b) Using the substitution $u = 2 - x$, or otherwise, find $\int_0^{\frac{1}{2}} \frac{x}{(2-x)^2} \, dx$ in the form $k - \ln \frac{4}{3}$, where k is a constant to be found. (6 marks)

5 A curve has equation $y = \frac{x^2 - 6}{2x - 5}$.

(a) Prove that there are no real values of x for which $2 < y < 3$. (6 marks)

(b) Hence find the coordinates of the two turning points on the curve. (3 marks)

(c) (i) State the equation of the vertical asymptote to the curve. (1 mark)

(ii) Given that $x^2 - 6 \equiv (2x - 5)\left(\frac{1}{2}x + \frac{5}{4}\right) + \frac{1}{4}$, find the equation of the oblique asymptote to the curve. (2 marks)

6 The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

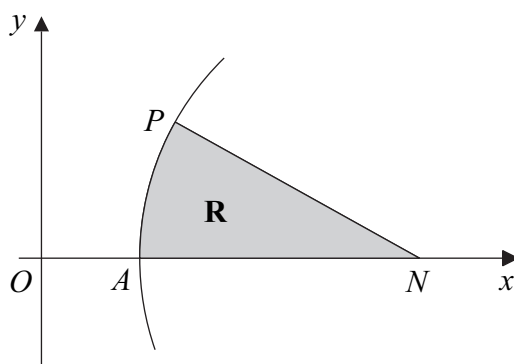
The line l_2 has equation $\mathbf{r} = \begin{pmatrix} -3 \\ 4 \\ 8 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$.

(a) Show that the lines l_1 and l_2 intersect and find the position vector of their point of intersection. (4 marks)

(b) The lines l_1 and l_2 lie in the plane Π . Write down a vector equation of Π in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$. (2 marks)

TURN OVER FOR THE NEXT QUESTION

7 The diagram shows part of a curve.



This curve is defined parametrically by

$$x = 4t + \frac{1}{t}, \quad y = 4t - \frac{1}{t}, \quad t > 0$$

The point P on the curve is where $t = 1$.

The normal to the curve at P intersects the x -axis at N .

The curve cuts the positive x -axis at the point A .

- (a) Show that $t = \frac{1}{2}$ at the point A . (2 marks)
- (b) Show that $\frac{dy}{dx} = \frac{4t^2 + 1}{4t^2 - 1}$. (2 marks)
- (c) (i) Find an equation of the normal PN . (3 marks)
(ii) Hence show that the x -coordinate of N is 10. (1 mark)
- (d) (i) Express $x + y$ and $x - y$ in terms of t . (2 marks)
(ii) Hence find a cartesian equation for the curve. (1 mark)
- (e) The region \mathbf{R} , bounded by the curve, the normal PN and the x -axis, is shown shaded in the diagram. Using your answer to part (d)(ii) and given that the area of \mathbf{R} is $15 - 8 \ln 2$, find the exact value of $\int_4^5 \sqrt{x^2 - 16} \, dx$. (3 marks)

END OF QUESTIONS