

General Certificate of Education  
January 2004  
Advanced Level Examination



**MATHEMATICS AND STATISTICS  
(SPECIFICATION B)  
Unit Pure 5**

**MBP5**

Monday 19 January 2004 Morning Session

**In addition to this paper you will require:**

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 15 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP5.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

**Information**

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

**Advice**

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

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- 1 Use the trapezium rule with five ordinates (four strips) to find an approximation to

$$\int_1^3 \frac{1}{x^3 + 3} dx$$

giving your answer to 3 significant figures.

(4 marks)

- 2 Given that  $y^3 + 3y = x^3$ , use implicit differentiation to show that

$$\frac{dy}{dx} = \frac{x^2}{y^2 + 1}$$

(3 marks)

- 3 (a) Obtain the first three terms of the binomial expansion of  $(1 + 4x^2)^{\frac{1}{2}}$  in ascending powers of  $x$ . (3 marks)
- (b) State the range of values of  $x$  for which the full expansion is valid. (2 marks)
- (c) By integrating the three terms in your expansion, find an approximate value for

$$\int_0^{\frac{1}{4}} (1 + 4x^2)^{\frac{1}{2}} dx$$

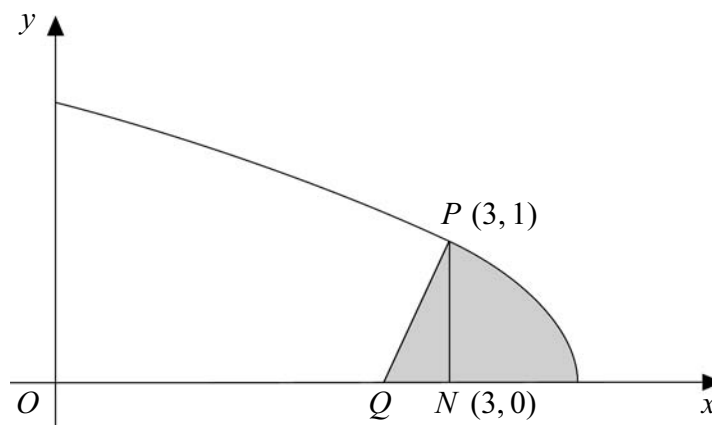
(3 marks)

- 4 (a) Express  $12 \sin 2x + 16 \cos 2x$  in the form  $R \sin(2x + \alpha)$ , where  $R$  is a positive constant and  $0 < \alpha < \frac{\pi}{2}$ . Give the value of  $\alpha$  to 3 significant figures. (3 marks)
- (b) A curve has equation  $y = 11x^2 - 3 \sin 2x - 4 \cos 2x$ .
- (i) Find  $\frac{d^2y}{dx^2}$ . (3 marks)
- (ii) Show that the curve does not have any points of inflection. (2 marks)

5 A curve has equation  $y = \frac{x^2}{x^2 + 3x + 3}$ .

- (a) Write down the equation of the horizontal asymptote to the curve. (1 mark)
- (b) (i) Prove that, for all real values of  $x$ ,  $y$  satisfies the inequality  $0 \leq y \leq 4$ . (6 marks)
- (ii) Hence find the coordinates of the turning points on the curve. (3 marks)
- (c) Given that there are no vertical asymptotes, sketch the curve. (3 marks)

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The diagram shows the curve  $C$  which is defined parametrically by

$$x = 4 \sin^2 t, \quad y = 2 \cos t, \quad 0 \leq t \leq \frac{\pi}{2}$$

The point  $P(3, 1)$  lies on the curve  $C$  and the foot of the perpendicular from  $P$  to the  $x$ -axis is  $N(3, 0)$ . The normal to the curve  $C$  at  $P$  intersects the  $x$ -axis at the point  $Q$ .

- (a) (i) Obtain an expression for  $\frac{dy}{dx}$  in terms of  $t$ . (3 marks)
- (ii) Find the value of  $t$  at the point  $P$ . (1 mark)
- (iii) Show that the equation of the normal  $PQ$  is  $y = 2x - 5$ . (3 marks)
- (b) (i) Show that the area of the region bounded by the curve  $C$ , the line  $PN$  and the  $x$ -axis is given by  $16 \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2 t \sin t \, dt$ . (3 marks)
- (ii) Using the substitution  $u = \cos t$ , or otherwise, evaluate  $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2 t \sin t \, dt$ . (3 marks)
- (iii) Hence find the area of the shaded region bounded by the curve  $C$ , the normal  $PQ$  and the  $x$ -axis. (2 marks)

7 The line  $l$  has equation  $\mathbf{r} = \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ .

The plane  $\Pi$  has equation  $\mathbf{r} \cdot \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = 5$ .

(a) Find the value of  $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ . *(1 mark)*

(b) Find the position vector of the point of intersection of  $l$  and  $\Pi$ . *(3 marks)*

(c) (i) Show that the angle between the vectors  $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$  and  $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$  is

$$\cos^{-1} \left( \frac{4\sqrt{70}}{35} \right) \quad \text{span style="float: right;">*(3 marks)*$$

(ii) Hence find, to the nearest degree, the angle between  $l$  and  $\Pi$ . *(2 marks)*

**END OF QUESTIONS**