GCE 2005
January Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics and Statistics B
(MBP5)

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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Key to Mark Scheme

M .... mark is for .... method
m.................................... mark is dependent on one or more M marks and is for .... method
A .... mark is dependent on M or m marks and is for .... accuracy
B .... mark is independent of M or m marks and is for .... method and accuracy
E .... mark is for .... explanation
√ or ft or F .... follow through from previous incorrect result
CAO .... correct answer only
AWFW... anything which falls within
AWRT.. anything which rounds to
AG .... answer given
SC .... special case
OE .... or equivalent
A2,1 .... 2 or 1 (or 0) accuracy marks
-*x* EE .... deduct *x* marks for each error
NMS .... no method shown
PI .... possibly implied
SCA .... substantially correct approach
c .... candidate
SF .... significant figure(s)
DP .... decimal place(s)

Abbreviations used in Marking

MC – *x* .... deducted *x* marks for mis-copy
MR – *x* .... deducted *x* marks for mis-read
ISW .... ignored subsequent working
BOD .... given benefit of doubt
WR .... work replaced by candidate
FB .... formulae booklet

Application of Mark Scheme

No method shown:
Correct answer without working .... mark as in scheme
Incorrect answer without working .... zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

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Question Number and Part	Solution	Marks	Total	Comments
1(a)	$y' = 2x - e^x$	B1		
	$y'' = 2 - e^x$	B1√	2	ft on slip
(b)	$y''' = -e^x$			
	$y^{(iv)} = -e^x$ ($\Rightarrow y''' = y^{(iv)}$ for all x)	B1	1	
(c)	$y'' = 0 \Rightarrow e^x = 2$	M1		Put $y'' = 0$ and a start
	$y''' = -e^x \neq 0$	B1		Check $y''' \neq 0$
	x-coord = ln 2 (=0.693(147..))	A1√		Only ft on one slip
	y-coord = $(\ln 2)^2 - 2$ (= −1.51(95..))	A1√	4	Only ft on one slip. Condone missing bracket if no contradiction
	Total		**7**	
2	$I \approx \frac{0.5}{3}\{\ldots.\}$	B1		Outside multiplier $\frac{0.5}{3}$.
	$\{\ldots\} = 1 + 4\sqrt{1.25} + 2\sqrt{2} + 4\sqrt{3.25} + \sqrt{5}$	M1		f(0)+4f(0.5)+2f(1)+4f(1.5)+f(2) attempted
	$I \approx \frac{0.5}{3}\begin{bmatrix}1 + 4(1.118\ldots.) + 2(1.414..) + \\ +4(1.8027\ldots.) + 2.236\ldots.\end{bmatrix}$			
	= 2.9579…	A1		
	To 3 dp the integral = 2.958	A1	4	cao Must be 2.958
	Total		**4**	
3	$2\sin x\cos x + \cos x = 0$ oe	M1		
	$\cos x = 0$ or $\sin x = -0.5$	A1		Either one
	$\cos x = 0 \Rightarrow x = 2n\pi \pm \ldots$ oe	m1		Condone degrees
	$\sin x = -0.5 \Rightarrow x = n\pi + (-1)^n\alpha$ oe	m1		Condone degrees
	$x = 2n\pi \pm \pi/2$ oe and $x = n\pi + (-1)^n(-\pi/6)$ oe	A1	5	Need both in rads. **sc** If m0m0 award B1 for four particular solutions 'covering all positions' or general solution(s) for two positions (condone degrees)
	Total		**5**	

MBP5 (cont)

Question Number and Part	Solution	Marks	Total	Comments
4(a)(i)	$(2-x)^{-2} = \left(2[1-\frac{x}{2}]\right)^{-2}$			
	$= 2^{-2}\left(1-\frac{x}{2}\right)^{-2} = \frac{1}{4}\left(1-\frac{x}{2}\right)^{-2}$	B1	1	**ag** Be convinced
(ii)	$\left(1-\frac{x}{2}\right)^{-2} \approx \left(1+(-2)\left(\frac{-x}{2}\right)+\frac{(-2)(-3)}{2!}\left(\frac{-x}{2}\right)^2+\ldots\right)$	M1		Condone $\frac{x}{2}$ in place of $-\frac{x}{2}$
	$= 1+x+\frac{3}{4}x^2+\ldots$	A1		Correct expansion and at least two of the three terms tidied correctly
	$(2-x)^{-2} = \frac{1}{4}\left(1+x+\frac{3}{4}x^2\right)$	A1	3	
(iii)	Valid for $-2 < x < 2$	B2,1	2	Condone use of modulus sign. B1 for reasonable attempt
(b)	$u = 2-x \Rightarrow \mathrm{d}u = -\,\mathrm{d}x$	B1		Accept $\frac{\mathrm{d}u}{\mathrm{d}x} = -1$ oe (possibly implied)
	$\ldots = \int \frac{(2-u)}{u^2}(-1\,\mathrm{d}u)$	M1		all x's and dx 'eliminated';
	$\ldots.. = \int \frac{1}{u} - \frac{2}{u^2}\,\mathrm{d}u$	m1		valid split of integrand oe
	$= \left[\ln u + \frac{2}{u}\right]$	m1		"[ ]", 2 terms at least one term correct…allow both negative
	$= \left[\ln u + \frac{2}{u}\right]_2^{\frac{3}{2}}$ =(ln1.5 + $\frac{4}{3}$) – (ln 2+1)	m1		Valid use of corresponding limits for u or a subst back to x with original limits used; dep only on 1st M but must have integrated
	$= \frac{1}{3}+\ln\frac{3}{4} = \frac{1}{3}-\ln\frac{4}{3}$	A1	6	cao be convinced
	Total		**12**	

MBP5 (cont)

Question Number and Part	Solution	Marks	Total	Comments
5(a)	$x^2 - 2yx + 5y - 6 \quad \{=0\}$	M1		Start to form quadratic in x with y involved
		A1		Correct quadratic in x
	$\Delta = (-2y)^2 - 4(1)(5y-6)$	m1		Considers b^2-4ac. Accept$(2y)^2$ for $(-2y)^2$
	…. $= 4(y^2 - 5y + 6)$	A1		If linked with 0, '4' may be omitted. Can be given even if a sign error causes prev. A0
	….. $= 4(y-2)(y-3)$	m1		Attempt to factorise or solve
	For no real x, $\Delta < 0 \Rightarrow 2 < y < 3$	A1	6	**ag** cso Be convinced. **NB** sign error in coeff of x in M1 line can earn max of M1A0m1A1m1A0
(b)	$y = 2 \Rightarrow x^2 - 4x + 4 = 0$ $y = 3 \Rightarrow x^2 - 6x + 9 = 0$	M1		Substitute $y = 2$ or $y = 3$ to form a valid quadratic in x.
	$\Rightarrow x = 2 \Rightarrow$ Turning point (2, 2)	A1		**sc** (Hence not used) Give correct answers B1 if no obvious errors in solution
	$\Rightarrow x = 3 \Rightarrow$ Turning point (3,3)	A1	3	
(c)(i)	Vert. asym. $x = \frac{5}{2}$	B1	1	
(ii)	$\frac{x^2-6}{2x-5} \equiv \frac{1}{2}x + \frac{5}{4} + \frac{\frac{1}{4}}{2x-5}$	M1		Division by $2x - 5$
	as $x \to \infty$, $y \to \frac{1}{2}x + \frac{5}{4}$			
	Oblique asymptote is $y = \frac{1}{2}x + \frac{5}{4}$	A1	2	
	Total		**12**	
6(a)	Intersect if $5 + s = -32t$, $3 + s = 4t$, $1 + s = 8 - 3t$	M1		Clear comparison to form two equations and attempt to solve
	Solving any two simultaneously gives	m1		Solving two eqns simultaneously as far as a value for s **or** a value for t
	$s = -2$ and $t = 3$ checking in 3rd eqn	A1		$s = -2$ and $t = 3$ **with** a valid check in a 3rd eqn.
	position vector of point of intersection is $\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$	B1	4	cao
(b)	$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$ oe	B2,1√	2	B1 if a small ft error
	Total		**6**	

MBP5 (cont)

Question Number and part	Solution	Marks	Total	Comments
7(a)	At A, $4t - \frac{1}{t} = 0$	M1		
	$\Rightarrow 4t^2 = 1 \Rightarrow t = \frac{1}{2}$	A1	2	**ag** Be convinced
(b)	$\frac{dx}{dt} = 4 - \frac{1}{t^2}$, $\frac{dy}{dt} = 4 + \frac{1}{t^2}$	M1		Attempts both and at least one correct (or both partially correct)
	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t^2+1}{4t^2-1}$	A1	2	**ag** Be convinced
(c)(i)	At P $t = 1 \Rightarrow \frac{dy}{dx} = \frac{5}{3}$			
	So gradient of the normal is $-\frac{3}{5}$	M1		Use of $m \times m' = -1$; must be constant
	P (5,3)	B1		
	Normal at P has eqn. $y - 3 = -\frac{3}{5}(x-5)$	A1√	3	Any correct form ft on one slip
(ii)	When $y = 0$, $x = 5 + 5 = 10$	A1	1	**ag** cao Be convinced
(d)(i)	$x + y = 8t$	B1		
	$x - y = \frac{2}{t}$	B1	2	
(ii)	Equation of C is $x^2 - y^2 = 16$ oe	B1√	1	ft only on answers pt and $\frac{q}{t}$ in part (d)(i)
(e)	Area of triangle NPP' = $\frac{1}{2}(10-5)(3) = 7.5$	B1		
	At A $x = 4$; at P $x = 5$ Area of **R** = $\int_4^5 y \, dx +$ area of triangle NPP′	M1		
	$\Rightarrow \int_4^5 \sqrt{x^2 - 16} \, dx = 7.5 - 8\ln 2$	A1	3	cso
	Total		**14**	
	TOTAL		**60**	