Mark scheme
January 2004

GCE

Mathematics & Statistics B

Unit MBP5

Key to mark scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m mark and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
– *x* EE		Deduct *x* marks for each error
NMS		No method shown
PI		Perhaps implied
c		Candidate

Abbreviations used in marking

MC – *x*	deducted *x* marks for miscopy
MR – *x*	deducted *x* marks for misread
ISW	ignored subsequent working
BOD	gave benefit of doubt
WR	work replaced by candidate

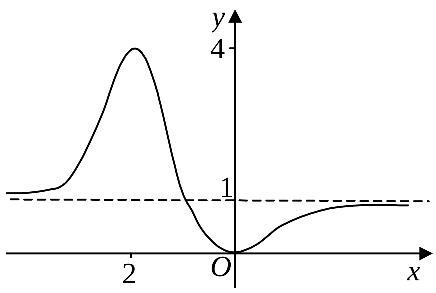
Application of mark scheme

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

Question Number and part	Solution	Marks	Total	Comments
1	$h = 0.5$	B1		
	Integral = $\frac{h}{2}\{\ldots\}$			
	$\{\ldots\}= \left[\frac{1}{4}+\frac{1}{30}+2\left(\frac{8}{51}+\frac{1}{11}+\frac{8}{149}\right)\right]$	M1 A1		At least 3 terms correct 5 terms, at least 4 correct
	Integral = 0.222	A1	4	cao must be 0.222
	sc (for 5 strips) $h = 0.4$			B0 M1 at least 4 terms correct A1 6 terms at least 5 correct A1cao
	Total		**4**	
2	$3y^2\frac{dy}{dx} + 3\frac{dy}{dx} = 3x^2$	M1 A1		2 terms correct
	$3\left(y^2+1\right)\frac{dy}{dx} = 3x^2$			
	$\Rightarrow \frac{dy}{dx} = \frac{x^2}{y^2+1}$	A1	3	**ag** cso
	Total		**3**	
3(a)	$\left(1+4x^2\right)^{\frac{1}{2}} \approx 1+\left(\frac{1}{2}\right)\left(4x^2\right)+\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(4x^2\right)^2}{2!}$	M1		Valid attempt to at least 2 terms
	$\ldots\ldots\ldots = 1+2x^2-2x^4+\ldots$	A2,1	3	A1 for correct expansion and at least 2 of 3 terms tidied correctly
(b)	$\lvert x \rvert < \frac{1}{2}$	B2,1	2	B1 for $\lvert 4x^2 \rvert < 1$ or better
(c)	Integral $\approx \int_0^{\frac{1}{4}} 1+2x^2-2x^4 \, dx$			
	$= \left[x+\frac{2}{3}x^3-\frac{2}{5}x^5\right]_0^{\frac{1}{4}}$	M1 A1√		Integrating 3 terms at least two integrated correctly ft on (a) if equivalent difficulty
	$= \frac{1}{4}+\frac{1}{96}-\frac{1}{2560}-0=\frac{1997}{7680} = 0.2600$ [0.25 + 0.0104(16..)−0.00039(06..)]	A1	3	Accept 0.26 provided clear evidence with no error
	Total		**8**	

Question Number and part	Solution	Marks	Total	Comments
4(a)	$R\cos\alpha = 12$ or $R\sin\alpha = 16$ or $\tan\alpha = \frac{16}{12}$	M1		Accept seen
	$R^2 = 12^2 + 16^2$	M1		Or use two of the three results in line1.
	$\Rightarrow$ 20 sin (2x +0.927)	A1	3	For α accept 0.927;0.295π; 53.1°or better. R must be 20.
(b)(i)	$\frac{dy}{dx} = 22x - 6\cos 2x + 8\sin 2x$	M1 A1		Sign/constant errors only oe
	$\frac{d^2y}{dx^2} = 22 + 12\sin 2x + 16\cos 2x$	A1√	3	oe ft on earlier slip (answer only gets 3/3)
(ii)	For pt. of inflection need $\frac{d^2y}{dx^2} = 0$ $22 + 12\sin 2x + 16\cos 2x = 0$	M1		Equates their $\frac{d^2y}{dx^2}$ to 0
	Need 22 + 20 sin (2x +0.927) = 0; not possible since sin$x \geq -1$ so no point of inflection	E1	2	**ag** adequate explanation based on sin (2x+α) = k where $\lvert k \rvert > 1$
	Total		**8**	

Question Number and part	Solution	Marks	Total	Comments
5(a)	$y = 1$	B1	1	Must be the equation
(b)(i)	$(y-1)x^2 + 3yx + 3y \quad \{=0\}$	M1 A1		Attempt to form quadratic in x Correct quadratic in x
	$\Delta = (3y)^2 - 4(y-1)(3y)$	m1		Considers $b^2 - 4ac$
	…. $-3y^2 + 12y$	A1		
	….. $-3y(y-4)$	m1		Attempt to factorise or solve
	For real x, $\Delta \geq 0 \Rightarrow 0 \leq y \leq 4$	A1	6	**ag** cso
(ii)	$y = 4 \Rightarrow 3x^2 + 12x + 12 = 0$	M1		Substitute $y = 4$ to form a 'valid' quadratic in x. (PI)
	$\Rightarrow x = -2$, turning point $(-2, 4)$	A1		If not using 'hence' then $(-2, 4)$ is B1 max.
	$\{y = 0 \Rightarrow -x^2 = 0 \Rightarrow x = 0\}$			
	Turning point (0,0)	B1	3	
(c)		B3,2,1	3	B1 for shape B1 for origin as only point where graph meets the axes B1 for correct behaviour at the 'end-points'
	Total		**13**	

Question Number and part	Solution	Marks	Total	Comments
6(a)(i)	$\frac{dx}{dt} = 8\sin t\cos t$, $\frac{dy}{dt} = -2\sin t$	M1 A1		Attempts both (PI) At least one correct
	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin t}{8\sin t\cos t} = \frac{-1}{4\cos t}$	A1	3	Any correct function of t
(ii)	At P, $t = \frac{\pi}{3}$	B1	1	Accept 3sf
(iii)	Gradient of normal at P = $\frac{4\cos\frac{\pi}{3}}{1}$	M1		Valid use of $mm' = -1$ to reach a constant gradient for the normal
	Normal at P has equation $y - 1 = 4\cos\frac{\pi}{3}(x-3)$	m1		Dependent on previous M
	$\Rightarrow y = 2x - 5$	A1	3	**ag** cso
(b)(i)	'Required area' = $\int y\frac{dx}{dt}\,dt$	M1		Need attempt to write integrand in terms of t. (ignore limits at this stage)
	….. = $\int 2\cos t(8\sin t\cos t)\,dt$	A1√		ft cand's $\frac{dx}{dt}$ if seen in (a)
	At P, $t = \frac{\pi}{3}$; at 'end-pt' $t = \frac{\pi}{2}$ So required area is given by $16\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\cos^2 t\sin t\ dt$	A1	3	**ag** cso
(ii)	$u = \cos t \Rightarrow du = -\sin t\ dt$	B1		Accept $\frac{du}{dt} = -\sin t$ oe (PI)
	… = $\int_{\frac{1}{2}}^{0} u^2(-du)$	M1		All x's and dx 'eliminated' and limits changed oe
	…. = $\frac{1}{24}$	A1	3	Condone inclusion of '16' if working with 16 times given integral
(iii)	Area of triangle $NPQ = \frac{1}{4}$	B1		
	Required area = $\frac{1}{4} + 16 \times \frac{1}{24} = \frac{11}{12}$	B1√	2	ft on 1 slip
	Total		**15**	

Question number and part	Solution	Marks	Total	Comments
7(a)	$\begin{pmatrix}1\\3\\2\end{pmatrix}\bullet\begin{pmatrix}0\\2\\1\end{pmatrix} = 0 + 6 + 2 = 8$	B1	1	
(b)	$\begin{pmatrix}4+t\\5+3t\\3+2t\end{pmatrix}\bullet\begin{pmatrix}0\\2\\1\end{pmatrix}=5$	M1		Elimination of **r** between l and Π (scalars on both sides)
	$10+6t+3+2t=5$	A1		
	$\Rightarrow t = -1 \Rightarrow$ position vector of pt. of intersection is (3**i** + 2**j** + **k**)	A1	3	Accept any correct form
(c)(i)	Magnitude of vectors are $\sqrt{14}$ and $\sqrt{5}$	B1		Award for one correct
	$8 = \sqrt{14} \times \sqrt{5}\cos\theta$	M1		Use of dot product (ft on earlier values)
	$\cos\theta=\frac{8}{\sqrt{70}}=\frac{8\sqrt{70}}{70}=\frac{4\sqrt{70}}{35}$	A1	3	cso condone answer left as $\cos\theta=\frac{4\sqrt{70}}{35}$
	$\Rightarrow \theta=\cos^{-1}\left(\frac{4\sqrt{70}}{35}\right)$			**ag**
(ii)	Angle between l and Π $= 90°-\theta$	M1		
	=72.967...=73°	A1	2	awrt 73°
	Total		**9**	
	TOTAL		**60**	