General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS AND STATISTICS **MBP4**
(SPECIFICATION B)
Unit Pure 4

Monday 21 June 2004 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 Differentiate each of the following with respect to x:

(a) $(1+x^2)^8$; *(2 marks)*

(b) $\dfrac{5x}{x^3+2}$. *(2 marks)*

2 The amount of money, £P, in a special savings account at time t years after 1st January 2000 is given by

$$P = 100 \times 1.05^t$$

(a) State the amount of money in the account on 1st January 2000. *(1 mark)*

(b) Calculate, to the nearest penny, the amount of money in the account on 1st January 2004. *(1 mark)*

(c) Find the value of t when $P = 150$, giving your answer to 3 significant figures. *(3 marks)*

3 (a) Express $\dfrac{13-2x}{(x+4)(2x+1)}$ in partial fractions. *(3 marks)*

(b) Hence, prove that

$$\int_0^4 \frac{13-2x}{(x+4)(2x+1)}\,dx = p\ln 3 - q\ln 2$$

where p and q are positive integers. *(4 marks)*

4 The polynomial $p(x)$ is given by

$$p(x) = x^3 - 6x^2 + 12x - 11$$

(a) Find the remainder when $p(x)$ is divided by $(x - 3)$. *(2 marks)*

(b) The equation $p(x) = 0$ has a single real root α.

(i) Show that α lies between 3 and 4. *(1 mark)*

(ii) Use the bisection method to find an interval of width 0.25 in which α lies. *(3 marks)*

(c) (i) Find the binomial expansion of $(x - 2)^3$. *(2 marks)*

(ii) Show that $p(x) = (x - 2)^3 - k$, stating the value of the constant k. *(1 mark)*

(iii) Hence find the exact solution of the equation $p(x) = 0$, leaving your answer in surd form. *(2 marks)*

5 A circle with centre $C(2, -5)$ has equation $x^2 + y^2 + ax + 10y = 7$, where a is a constant.

(a) (i) Find the value of a. *(2 marks)*

(ii) Show that the radius of the circle is 6 units. *(2 marks)*

(b) The line l_1 has equation $24x + 7y = 5k + 3$, where k is a constant.

(i) Prove that the distance from C to l_1 is $\dfrac{|2 - k|}{5}$. *(2 marks)*

(ii) The line l_1 intersects the circle. Show that

$$|2 - k| \leqslant 30$$

Hence find the range of values satisfied by k. *(4 marks)*

6 (a) Prove the identity

$$(3\sin x + 5\cos x)^2 \equiv 17 + 8\cos 2x + 15\sin 2x$$ *(4 marks)*

(b) Hence find:

(i) $\int (3\sin x + 5\cos x)^2 \, dx$; *(3 marks)*

(ii) the volume of the solid formed when the region bounded by the curve with equation $y = 3\sin x + 5\cos x$, the coordinate axes and the line $x = \frac{\pi}{4}$ is rotated through 2π radians about the x-axis. *(3 marks)*

(c) (i) Show that the equation

$$(3\sin x + 5\cos x)^2 = 4\cos^2 x$$

can be written in the form

$$(3\tan x + 5)^2 = 4$$ *(1 marks)*

(ii) Hence, or otherwise, solve the equation

$$(3\sin x + 5\cos x)^2 = 4\cos^2 x$$

giving all solutions in radians in the interval $0 < x < \pi$. *(5 marks)*

7 The function f is defined for $0 < x < \frac{\pi}{3}$ by $f(x) = 5\,\text{cosec}\,3x$.

(a) Find $f\left(\frac{\pi}{4}\right)$ in the form $p\sqrt{2}$. *(2 marks)*

(b) (i) Find the derivative $f'(x)$. *(2 marks)*

(ii) Hence find $f'\left(\frac{\pi}{4}\right)$ in the form $q\sqrt{2}$. *(2 marks)*

(c) Find the equation of the tangent to the curve with equation $y = f(x)$ at the point where $x = \frac{\pi}{4}$. *(1 mark)*

END OF QUESTIONS