

General Certificate of Education
January 2005
Advanced Level Examination



**MATHEMATICS AND STATISTICS
(SPECIFICATION B)
Unit Pure 4**

MBP4

Tuesday 25 January 2005 Morning Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A curve has equation

$$y = \frac{4x}{\sin x}, \quad 0 < x < \pi$$

(a) Find the value of $\frac{dy}{dx}$ when $x = \frac{\pi}{2}$. (4 marks)

(b) Use the result from part (a) to find the approximate change in y when x increases from $\frac{\pi}{2}$ to $\frac{\pi}{2} + 0.01$. (2 marks)

2 The polynomial $p(x)$ is given by $p(x) = x^3 - 3x + 2$.

(a) Find the remainder when $p(x)$ is divided by $x + 1$. (2 marks)

(b) Given that $x + 2$ is a factor of $p(x)$, express $p(x)$ as a product of linear factors. (3 marks)

(c) Simplify the following algebraic fraction as far as possible:

$$\frac{x^3 - 3x + 2}{x^2 - 1} \quad (2 \text{ marks})$$

3 A circle has equation $x^2 + y^2 - 6x + 10y + 18 = 0$.

(a) (i) Find the coordinates of the centre, C . (2 marks)

(ii) Find the radius of the circle. (2 marks)

(b) Explain why the circle lies entirely below the x -axis. (2 marks)

(c) The point P has coordinates $(11, 6)$. Find the length of CP and hence find the length of a tangent from P to the circle. (3 marks)

- 4 (a) (i) Express $\sin 2\theta$ in terms of $\sin \theta$ and $\cos \theta$. (1 mark)
- (ii) Express $\cos 2\theta$ in terms of $\cos \theta$. (1 mark)
- (iii) Prove the identity

$$\tan \theta(1 + \cos 2\theta) \equiv \sin 2\theta \quad (2 \text{ marks})$$

- (b) Solve the equation

$$\tan \theta(1 + \cos 2\theta) = 2 \sin^2 2\theta$$

for $0 < \theta < \pi$, leaving your answers in terms of π . (6 marks)

- 5 (a) Solve the equation $3^x = 7$, giving your answer to three significant figures. (2 marks)
- (b) (i) Sketch the graphs of $y = 3^x$ and $y = 7 - x^2$ on the same axes. (2 marks)
- (ii) Hence state the number of roots of the equation $3^x = 7 - x^2$. (1 mark)
- (c) (i) Show that the equation $3^x = 7 - x^2$ can be written in the form

$$x = \frac{\ln(7 - x^2)}{\ln 3} \quad (1 \text{ mark})$$

- (ii) Use the iterative formula

$$x_{n+1} = \frac{\ln(7 - x_n^2)}{\ln 3}, \quad x_1 = 1.5$$

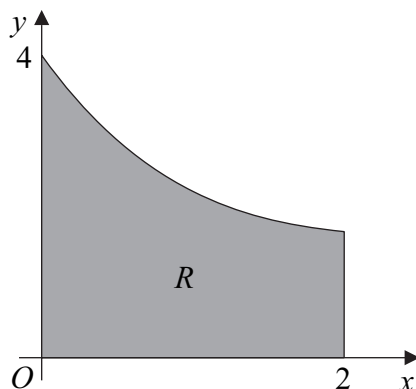
to find the values of x_2 and x_3 , giving your answers to three decimal places.

(2 marks)

TURN OVER FOR THE NEXT QUESTION

- 6 A curve has equation $y = 1 + \frac{6}{3x + 2}$ and is sketched below for $0 \leq x \leq 2$.

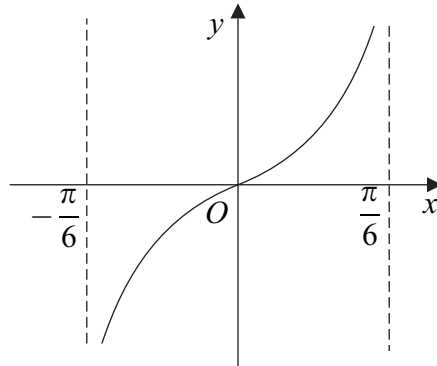
The shaded region R is bounded by the curve, the coordinate axes and the line $x = 2$.



- (a) Express y^2 in the form $1 + \frac{A}{3x + 2} + \frac{B}{(3x + 2)^2}$. (2 marks)
- (b) Find:
- (i) $\int \frac{1}{3x + 2} dx$; (2 marks)
- (ii) $\int \frac{1}{(3x + 2)^2} dx$. (2 marks)
- (c) Find the volume of the solid formed when R is rotated completely about the x -axis, giving your answer to three significant figures. (5 marks)

- 7 (a) The function f is defined for $-\frac{\pi}{6} < x < \frac{\pi}{6}$ by $f(x) = \tan 3x$.

The graph of $y = f(x)$ is sketched below.



The inverse of f is f^{-1} .

- (i) Sketch the graph of $y = f^{-1}(x)$. *(2 marks)*
- (ii) Find an expression for $f^{-1}(x)$. *(2 marks)*
- (b) A curve has equation $x = \tan 3y$.
- (i) Find $\frac{dx}{dy}$ in terms of y . *(2 marks)*
- (ii) Hence find the value of $\frac{dy}{dx}$ when $y = \frac{\pi}{9}$. *(3 marks)*

END OF QUESTIONS

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