

General Certificate of Education
January 2004
Advanced Level Examination



**MATHEMATICS AND STATISTICS
(SPECIFICATION B)
Unit Pure 4**

MBP4

Friday 23 January 2004 Morning Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The population, P , of insects in a colony is given by $P = Ae^{kt}$, where A and k are constants and the time t is measured in months.

(a) Given that $P = 500$ when $t = 0$ and that $P = 750$ when $t = 10$, find the value of k .
(3 marks)

(b) Find the value of t when $P = 1500$, giving your answer to 3 significant figures.
(2 marks)

2 A model plane moves so that its height, y metres, above horizontal ground is given by

$$y = \frac{8x}{x^3 + 1}, \quad x \geq 0$$

when its horizontal distance from the take-off point on the ground is x metres.

(a) Find the value of $\frac{dy}{dx}$ when $x = 1$.
(3 marks)

(b) (i) Find the rate of change of y in m s^{-1} when $x = 1$ and x is increasing at a rate of 0.8 m s^{-1} .
(2 marks)

(ii) Interpret the sign in your answer to part (i).
(1 mark)

3 The polynomial $p(x)$ is given by

$$p(x) = (x + 3)(x - 2)(x - 4)$$

(a) Find the remainder when $p(x)$ is divided by $(x + 1)$.
(2 marks)

(b) (i) Express $\frac{70}{(x + 3)(x - 2)(x - 4)}$ in the form $\frac{A}{x + 3} + \frac{B}{x - 2} + \frac{C}{x - 4}$.
(3 marks)

(ii) Hence, prove that $\int_5^6 \frac{70}{(x + 3)(x - 2)(x - 4)} dx = N \ln 3 - M \ln 2$,
where N and M are positive integers.
(5 marks)

4 A circle has equation $x^2 + y^2 - 10x - 6y + \frac{111}{4} = 0$.

(a) (i) Find the coordinates of the centre, C . (2 marks)

(ii) Find the radius of the circle. (2 marks)

(b) The line l_1 has equation $3x - 4y - 16 = 0$.

(i) Find the distance from C to l_1 . (2 marks)

(ii) Hence determine whether the line l_1 intersects the circle. (1 mark)

(iii) The line l_2 has equation $y = 2x + 5$.

Show that the acute angle between l_1 and l_2 is $\tan^{-1} \frac{1}{2}$. (3 marks)

5 A sequence is defined by

$$x_{n+1} = \sqrt{(x_n + 12)}, \quad x_1 = 2$$

(a) Find the values of x_2 , x_3 , and x_4 , giving your answers to 3 decimal places. (3 marks)

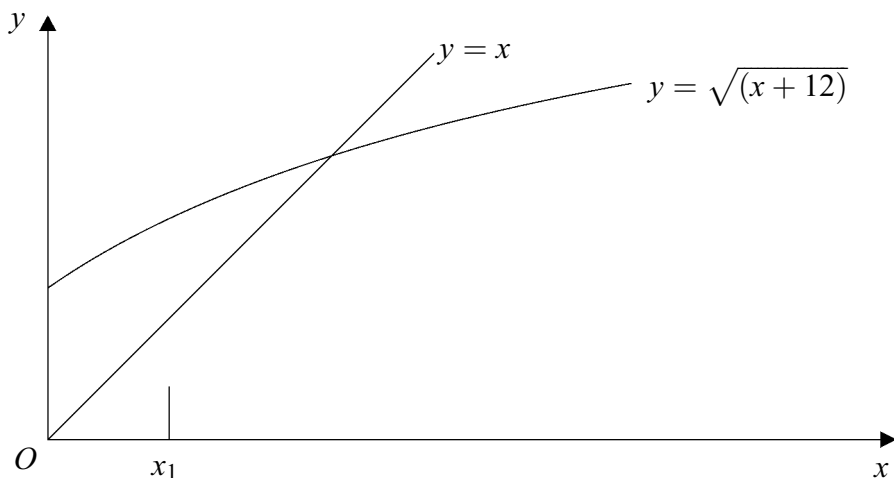
(b) Given that the limit of the sequence is L :

(i) show that L must satisfy the equation $L^2 - L - 12 = 0$; (2 marks)

(ii) find the value of L . (2 marks)

(c) The graphs of $y = \sqrt{(x + 12)}$ and $y = x$ are sketched below.

On a copy of the sketch, draw a cobweb or staircase diagram to show how convergence takes place.



(2 marks)

6 (a) Use the identity

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

to show that the equation

$$\cos\left(x + \frac{5\pi}{6}\right) = \sin x$$

can be written as

$$\cos x + \sqrt{3} \sin x = 0 \quad (4 \text{ marks})$$

(b) Hence solve the equation

$$\cos\left(x + \frac{5\pi}{6}\right) = \sin x$$

giving all solutions, in terms of π , in the interval $0 < x < 2\pi$. (3 marks)

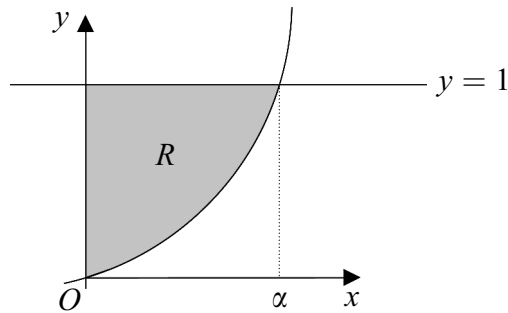
7 A curve is defined for $0 \leq x < \frac{\pi}{4}$ by the equation $y = \tan 2x$.

(a) (i) Find $\frac{dy}{dx}$. (2 marks)

(ii) Hence, find the equation of the tangent to the curve at the point $\left(\frac{\pi}{6}, \sqrt{3}\right)$. (2 marks)

(b) Find $\int (\sec^2 2x - 2 \tan 2x) dx$. (3 marks)

(c) The curve $y = \tan 2x$ intersects the line $y = 1$ when $x = \alpha$, as shown in the diagram.



(i) Find the value of α in terms of π . (1 mark)

(ii) The shaded region bounded by the curve, the y -axis and the line $y = 1$ is R .

The region R is rotated through 2π radians about the line $y = 1$.

Show that the volume of the solid of revolution, V , is given by

$$V = \pi \int_0^{\alpha} (\sec^2 2x - 2 \tan 2x) dx \quad (3 \text{ marks})$$

(iii) Prove that $V = \frac{\pi}{2}(1 - \ln 2)$. (2 marks)

END OF QUESTIONS