

GCE 2004

June Series



Mark Scheme

Mathematics and Statistics B

MBP4

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Publications Department, Aldon House, 39, Heald Grove, Rusholme, Manchester, M14 4NA
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Dr Michael Cresswell Director General

Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
✓ or ft or F		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
-x ee		deduct x marks for each error
pi		possibly implied
sca		substantially correct approach

Abbreviations used in Marking

MC – x	deducted x marks for mis-copy
MR – x	deducted x marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

Application of Mark Scheme

No method shown:	
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

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Question Number and Part	Solution	Marks	Total	Comments
1(a)	$16x(1+x^2)^7$	M1 A1	2	*** $(1+x^2)^7$ for M1
(b)	$\frac{5(x^3+2)-5x(3x^2)}{(x^3+2)^2} \left[= \frac{10(1-x^3)}{(x^3+2)^2} \right]$	M1 A1	2	Quotient rule or product rule with negative power – condone one slip Correct unsimplified
Total			4	
2 (a)	£ 100	B1	1	
(b)	£121.55	B1	1	$P = 100 \times 1.215506\dots$
(c)	$1.05^t = 1.5$ $\Rightarrow t \ln 1.05 = \ln 1.5$ $\Rightarrow t = 8.31$	B1 M1 A1	3	Taking logs to base e or 10 Condone more SF rounding to 8.31
Total			5	
3(a)	$\frac{A}{x+4} + \frac{B}{2x+1}$ $A = -3, B = 4$	M1 m1 A1	3	Correct split Comparing coeffs, sub'n etc One correct value may imply m1
(b)	$A \ln(x+4) + \frac{1}{2} B \ln(2x+1)$ $(-3 \ln 8 + 2 \ln 9) - (-3 \ln 4)$ $= 4 \ln 3 - 3 \ln 2$	B1✓ B1✓ M1 A1	4	ft their A and B ft $-3 \ln(x+4) + 2 \ln(2x+1)$ Sub of limits 0 and 4, (must have more than 2 ln terms for M1) ($p = 4, q = 3$)
Total			7	
4(a)	$p(3) = 27 - 54 + 36 - 11$ $= -2$ (is remainder)	M1 A1	2	Must consider $p(3)$ or full long division to remainder
(b)(i)	$p(4) = 64 - 96 + 48 - 11 = 5$ [Change of sign] $\Rightarrow \alpha$ lies between 3 and 4	B1	1	Both $p(3)$ and $p(4)$ must be correct and there must be some statement/conclusion
(ii)	$p(3.5)$ used first (=0.375) $p(3.25) = -1.046875$ \Rightarrow root lies between 3.25 and 3.5	M1 m1 B1	3	\Rightarrow root lies between 3 and 3.5
(c)(i)	$x^3 + 3 \times (-2)x^2 + 3 \times (-2)^2 x + (-2)^3$ $= x^3 - 6x^2 + 12x - 8$	M1 A1	2	Attempt at row 1 3 ... of Pascal's triangle or (binomial) expansion All correct and simplified
(ii)	$p(x) = (x-2)^3 - 3$	B1	1	($k = 3$)
(iii)	$x-2 = \sqrt[3]{3}$ $\Rightarrow x = 2 + \sqrt[3]{3}$	M1 A1✓	2	Attempt to isolate x ft from (ii)
Total			11	

MBP4 (cont)

Question Number and Part	Solution	Marks	Total	Comments
5(a)(i)	$(x-2)^2 + (y+5)^2 = r^2$	M1		Or attempt to complete square etc
	$a = -4$	A1	2	Correct answer implies M1.
(ii)	$r^2 = 7 + 2^2 + 5^2 = 36$	M1		Attempt at r^2 from formula or equation
	$\Rightarrow r = 6$	A1	2	ag be convinced – watch circular arg't
(b)(i)	$d = \frac{ 24 \times 2 + 7 \times (-5) - (5k + 3) }{\sqrt{7^2 + 24^2}}$	M1		Use of formula for distance, condone a sign slip or omission of brackets or mod signs
	$= \frac{ 10 - 5k }{25}$			
	$= \frac{ 2 - k }{5}$	A1	2	ag (algebra must follow correctly) condone no modulus signs
(ii)	$\frac{ 2 - k }{5} \leq 6 \Rightarrow 2 - k \leq 30$	B1		ag be convinced they are using radius of circle
	Boundaries for k are -28 and 32	B1		
	$-28 \leq k \leq 32$	B1	4	Allow $-28 < k < 32$
	Total		10	
6(a)	$9\sin^2 x + 30\sin x \cos x + 25\cos^2 x$	B1		ag be convinced
	$30\sin x \cos x = 15\sin 2x$	B1		
	Use of $\cos 2x = 2\cos^2 x - 1$ (or $1 - 2\sin^2 x$)	M1		
	$9 + 16\cos^2 x = 17 + 8\cos 2x$	A1	4	ag be convinced
(b)(i)	$17x + 4\sin 2x - \frac{15}{2}\cos 2x \quad (+C)$	M1		Attempt to integrate a trig term
		A1		One trig term correct
		A1	3	All correct (condone no +C)
(ii)	$\pi \int_0^{\frac{\pi}{4}} y^2 dx$	B1		May be simply stated and not used
	$= \pi \left(\frac{17}{4}\pi + \frac{23}{2} \right)$	M1		Sub limits into their (i) answer (condone missing π for M1)
		A1	3	Or equivalent $78.07\dots, 24.85\pi, \text{etc}$
(c)(i)	$(3\sin x + 5\cos x)^2 = 4\cos^2 x$			
	$\Rightarrow \left(3\frac{\sin x}{\cos x} + 5 \right)^2 = 4 \Rightarrow (3\tan x + 5)^2 = 4$	B1	1	ag be convinced
(ii)	$3\tan x + 5 = 2$	M1		
	$\Rightarrow \tan x = -1$	A1		
	$\Rightarrow x = \frac{3\pi}{4}$	A1		Condone $2.356\dots$ radians or 135°
	$3\tan x + 5 = -2 \Rightarrow \tan x = -\frac{7}{3}$	M1		Use of negative root
	$\Rightarrow x = 1.976\dots^c$	A1	5	Withhold final A1 if both answers not in radians or extra solutions given
	Total		16	

MBP4 (cont)

Question Number and Part	Solution	Marks	Total	Comments
7(a)	$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ $f\left(\frac{\pi}{4}\right) = 5\sqrt{2}$	M1 A1	2	Evidenced by $\operatorname{cosec}\left(\frac{3\pi}{4}\right) = \sqrt{2}$ (or 1.414...) or final answer 7.07106... Must be exactly this
(b) (i)	$f'(x) = -15\operatorname{cosec} 3x \cot 3x$	M1 A1	2	$k \operatorname{cosec}^{***} \cot^{***}$
(ii)	either $\operatorname{cosec}\left(\frac{3\pi}{4}\right) = \sqrt{2}$ or $\cot\left(\frac{3\pi}{4}\right) = -1$ $f'(x) = -15 \times \sqrt{2} \times (-1) = 15\sqrt{2}$	M1 A1	2	Must be exactly this
(c)	$y - 5\sqrt{2} = 15\sqrt{2}\left(x - \frac{\pi}{4}\right)$	B1✓	1	ft y – their $f\left(\frac{\pi}{4}\right) =$ their $f'\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right)$ Must involve surds
	Total		7	
	TOTAL		60	