GCE 2005
January Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics and Statistics B
(MBP4)

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

Set and published by the Assessment and Qualifications Alliance.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales 3644723 and a registered charity number 1073334. Registered address AQA, Devas Street, Manchester. M15 6EX. *Dr Michael Cresswell Director General*

Key to Mark Scheme

M ..... mark is for ..... method
m ..... mark is dependent on one or more M marks and is for ..... method
A ..... mark is dependent on M or m marks and is for ..... accuracy
B ..... mark is independent of M or m marks and is for ..... method and accuracy
E ..... mark is for ..... explanation
√ **or ft or F** ..... follow through from previous incorrect result
CAO ..... correct answer only
AWFW ..... anything which falls within
AWRT ..... anything which rounds to
AG ..... answer given
SC ..... special case
OE ..... or equivalent
A2,1 ..... 2 or 1 (or 0) accuracy marks
-*x* EE ..... deduct *x* marks for each error
NMS ..... no method shown
PI ..... possibly implied
SCA ..... substantially correct approach
c ..... candidate
SF ..... significant figure(s)
DP ..... decimal place(s)

Abbreviations used in Marking

MC – *x* ..... deducted *x* marks for mis-copy
MR – *x* ..... deducted *x* marks for mis-read
ISW ..... ignored subsequent working
BOD ..... given benefit of doubt
WR ..... work replaced by candidate
FB ..... formulae booklet

Application of Mark Scheme

No method shown:
Correct answer without working ..... mark as in scheme
Incorrect answer without working ..... zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

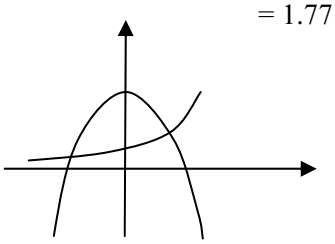
Crossed out work — do not mark unless it has not been replaced

Alternative solution using a correct or partially correct method — award method and accuracy marks as appropriate

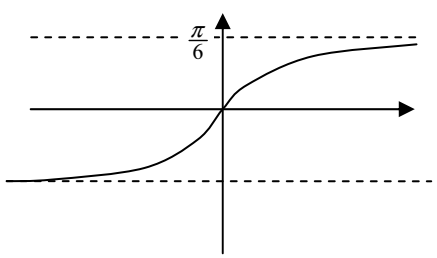
Mathematics and Statistics B Pure 4 MBP4 January 2005

Question Number and Part	Solution	Marks	Total	Comments
1(a)	$\frac{d(\sin x)}{dx} = \cos x$	B1		or $\frac{d(\text{cosec } x)}{dx} = -\text{cosec } x \cot x$
	$\frac{dy}{dx} = \frac{4\sin x - 4x\cos x}{(\sin x)^2}$	M1		quotient rule, or product rule, attempt (condone sign errors)
		A1		correct
	(oe $\frac{dy}{dx} = 4\,\text{cosec } x - 4x\,\text{cosec } x \cot x$)			
	$x = \frac{\pi}{2} \quad \Rightarrow \frac{dy}{dx} = 4$	A1√	4	
(b)	$\delta y \approx \frac{dy}{dx} \times \delta x$	M1		Stated or used – but NOT $\frac{dy}{dt} = \ldots$ etc
	$= 0.04$	A1√	2	ft their $\frac{dy}{dx}$ with $\delta x = 0.01$ Correct answer may score M0
	Total		**6**	
2(a)	p(–1) = –1+3+2	M1		p(–1) = ... or long division to remainder
	(remainder) = 4	A1	2	
(b)	Attempt at quadratic or long division	M1		or another linear factor
	$(x+2)(x^2-2x+1)$	A1		Or $(x + 2)(x – 1)$
	$= (x+2)(x-1)^2$	A1	3	
(c)	$\frac{(x+2)(x-1)^2}{(x+1)(x-1)}$	M1		Their (b) and denominator factorised
	$= \frac{(x+2)(x-1)}{(x+1)}$	A1	2	Withhold if further incorrect cancelling
	Total		**7**	
3(a)(i)	$(x-3)^2 + (y+5)^2$	M1		Attempt at completing square (generous) or one coordinate correct
	Centre (3, –5)	A1	2	
(ii)	$r^2 = 9 + 25 - 18 = 16$	M1		3 numbers - condone sign error
	$r = 4$	A1	2	
(b)	$\lvert y_C \rvert > 4$	E1		Or no real roots when $y = 0$
	Centre below x-axis	E1	2	
(c)	$CP^2 = 8^2 + 11^2 \quad \Rightarrow CP = \sqrt{185}$	B1√		ft their C
	$PT^2 = CP^2 - r^2$	M1		
	$PT^2 = 185 - 16 = 169 \quad \Rightarrow PT = 13$	A1√	3	Ft their CP & r provided $PT^2 > 0$
	Total		**9**	

MBP4 (cont)

Question Number and Part	Solution	Marks	Total	Comments
4(a)(i)	$2\sin\theta\cos\theta$	B1	1	
(ii)	$2\cos^2\theta - 1$	B1	1	
(iii)	$\tan\theta(2\cos^2\theta)$	M1		Sub their "$\cos 2\theta$" expression
	$= 2\sin\theta\cos\theta = \sin 2\theta$	A1	2	**ag** be convinced
(b)	$\sin 2\theta = 2\sin^2 2\theta$	B1		
	$\sin 2\theta = 0$	M1		
	$\Rightarrow \theta = \frac{\pi}{2}$	A1		Accept 1.57 ... or 90° Ignore 0, π or any values outside interval
	$\sin 2\theta = \frac{1}{2}$	M1		$\Rightarrow 2\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \ldots$
	$\Rightarrow \theta = \frac{\pi}{12}$	A1		accept 0.083π or better accept 15° or 75° if A1 not awarded for 90°
	$\Rightarrow \theta = \frac{5\pi}{12}$	A1	6	accept 0.417π or better (NOT 0.416π) All 3 must be correct and in terms of π and no extra solutions for final A1
	Total		**10**	
5(a)	$x = \frac{\ln 7}{\ln 3}$ $\left(or \frac{\log_{10} 7}{\log_{10} 3}\right)$	M1		
	$= 1.77$	A1	2	condone more sf 1.77124....
(b)(i)		B1		$y = 3^x$ general shape
		B1	2	$y = 7 - x^2$ general shape
(ii)	2 roots	B1√	1	ft their graphs
(c)(i)	$x\ln 3 = \ln(7 - x^2) \Rightarrow x = \frac{\ln(7 - x^2)}{\ln 3}$	B1	1	**ag** be convinced
(ii)	$x_2 = 1.418$	B1		accept 1.42 or more SF 1.418284...
	$x_3 = 1.463$	B1	2	Must be 3 dp
	Total		**8**	

MBP4 (cont)

Question Number and Part	Solution	Marks	Total	Comments
6(a)	$y^2 = 1 + \frac{12}{3x+2} + \frac{36}{(3x+2)^2}$	M1 A1	2	3 non-zero terms and 2 of them correct all correct
(b)(i)	$\int \frac{1}{3x+2}\,dx = \frac{1}{3}\ln(3x+2)$	M1 A1	2	$k\ln(3x+2)$ correct – condone missing +constant
(ii)	$\int \frac{1}{(3x+2)^2}\,dx = -\frac{1}{3}(3x+2)^{-1}$	M1 A1	2	$k(3x+2)^{-1}$ correct – condone missing +constant
(c)	$\pi\int_0^2 y^2 dx$	B1		
	$\left[x + 4\ln(3x+2) - 12(3x+2)^{-1}\right]$	M1 A1		Attempt to integrate "their" y^2 (2 terms) Correct unsimplified
	$(\pi)\left[2 + 4\ln(8/2) - 12(1/8 - 1/2)\right]$	m1		Evaluation of limits F(2) – F(0) (3 terms)
	$= 37.8$	A1	5	37.8410409...(condone more figures)
	Total		**11**	
7(a)(i)		B1 B1	2	General shape reflected in $y = x$ Asymptotes $y = \pm\frac{\pi}{6}$; (may be implied by numbers on y-axis); gradient > 0
(ii)	$y = \tan 3x \Rightarrow x = \frac{1}{3}\tan^{-1} y$	M1		Good attempt at x = ...
	$f^{-1}(x) = \frac{1}{3}\tan^{-1} x$	A1	2	
(b)(i)	$\frac{dx}{dy} = 3\sec^2 3y$	M1 A1	2	$k\sec^2$ ** correct
(ii)	$\frac{dy}{dx} = 1 \div their\ \frac{dx}{dy}$	M1		
	$= \frac{1}{3\sec^2 3y}$	A1		or $\frac{1}{3\sec^2(\pi/3)}$
	When $y = \frac{\pi}{9}$, $\frac{dy}{dx} = \frac{1}{12}$	A1	3	Accept 0.083 (or better) if all working shown
	Total		**9**	
	TOTAL		**60**	