General Certificate of Education
June 2005
Advanced Subsidiary Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) **MBP3**
Unit Pure 3

Wednesday 22 June 2005 Afternoon Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 45 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) A curve has equation $y = \dfrac{4x-3}{2-x}$.

(i) Find the coordinates of the points where the curve crosses the coordinate axes. *(2 marks)*

(ii) State the equations of its asymptotes. *(2 marks)*

(iii) Sketch the curve. *(2 marks)*

(b) Hence, or otherwise, solve the inequality

$$\frac{4x-3}{2-x} < 1$$ *(3 marks)*

2 (a) The roots of the quadratic equation $x^2 - 3x + 5 = 0$ are α and β.

(i) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$. *(2 marks)*

(ii) Without solving the quadratic equation, find the value of $\alpha^2 + \beta^2$.

Hence explain why α and β cannot both be real. *(3 marks)*

(iii) Show that $\alpha^3 + \beta^3 = -18$. *(2 marks)*

(b) Determine a quadratic equation with integer coefficients which has roots $\dfrac{\alpha^2}{\beta}$ and $\dfrac{\beta^2}{\alpha}$. *(4 marks)*

3 The matrix $\boldsymbol{M}$ is $\begin{bmatrix} k & -7 \\ -2 & 5 \end{bmatrix}$. The transformation **T** is given by $\begin{bmatrix} x' \\ y' \end{bmatrix} = \boldsymbol{M}\begin{bmatrix} x \\ y \end{bmatrix}$.

(a) Find the determinant of $\boldsymbol{M}$, giving your answer in terms of k. *(1 mark)*

(b) A quadrilateral is transformed by **T** into a quadrilateral with the same area as the original. Find the two values of k for which this occurs. *(3 marks)*

(c) When $k = 4$:

(i) find the inverse matrix $\boldsymbol{M}^{-1}$; *(2 marks)*

(ii) find the coordinates of the point which is mapped onto $(1, 7)$ under **T**. *(3 marks)*

4 (a) Find the complex roots of the quadratic equation

$$x^2 - 4x + 13 = 0$$ *(3 marks)*

(b) Given that $(p + 3\mathrm{i})^2 = q + 12\mathrm{i}$, find the value of each of the real numbers p and q. *(3 marks)*

5 Prove by mathematical induction that, for all positive integers n,

$$\sum_{r=1}^{n} \frac{r}{2^r} = 2 - \frac{(n+2)}{2^n}$$ *(5 marks)*

6 (a) A curve has equation

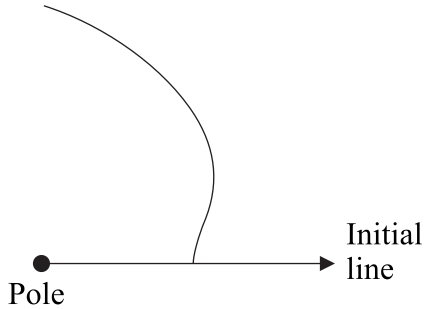
$$r = 3 - 2\cos\theta$$

for $-\pi < \theta \leqslant \pi$, where $[r, \theta]$ are polar coordinates.

(i) State the maximum and minimum values of r.

For each of these values of r, give a corresponding value of θ. *(4 marks)*

(ii) The curve is shown below for $0 \leqslant \theta \leqslant \frac{\pi}{2}$. Sketch the curve for $-\pi < \theta \leqslant \pi$.



(2 marks)

(b) (i) Find the values of $\cos\theta$ for which

$$8\cos^2\theta = 3 - 2\cos\theta$$ *(2 marks)*

(ii) Find the polar coordinates of the points of intersection of the curve with equation $r = 3 - 2\cos\theta$ and the curve with equation $r = 8\cos^2\theta$. *(4 marks)*

7 The value of a car, £V, can be modelled by the equation

$$V = 15\,000 - 5\,000\sqrt{t}\ , \qquad 0 \leqslant t \leqslant 8$$

where t is the time, in years, measured from when the car is new.

(a) Use this model to find the value of the car when:

(i) it is new; *(1 mark)*

(ii) it is 4 years old. *(1 mark)*

(b) (i) Find the value of $\dfrac{\mathrm{d}V}{\mathrm{d}t}$ when $t = 4$. *(3 marks)*

(ii) Interpret your answer to part (b)(i) in the context of this model. *(2 marks)*

(c) A second model relating V and t is proposed.

It is given by $V = a \times b^{-t}$, where a and b are constants.

(i) Express $\log_{10} V$ in terms of $\log_{10} a$, $\log_{10} b$ and t. *(2 marks)*

(ii) Given that $V = 11\,500$ when $t = 1$ and $V = 5\,000$ when $t = 4$, find the value of a and the value of b, giving your answers to three significant figures. *(4 marks)*

8 The binary operation $\otimes$ is defined by $p \otimes q = p + q + pq$, where p and q are real numbers.

(a) Show that the identity element is 0. *(1 mark)*

(b) Find the inverse of 3. *(3 marks)*

(c) (i) Express $(p \otimes q) \otimes r$ in a form not involving $\otimes$. *(2 marks)*

(ii) Hence determine whether $\otimes$ is associative. *(2 marks)*

9 (a) Sketch the graph of $\dfrac{x^2}{4} - \dfrac{y^2}{9} = 1$, marking clearly the values of any intercepts with the coordinate axes. *(3 marks)*

(b) Describe the geometrical transformation that maps the graph of $\dfrac{x^2}{4} - \dfrac{y^2}{9} = 1$ onto the graph of $\dfrac{(x+1)^2}{4} - \dfrac{y^2}{36} = 1$. *(4 marks)*

END OF QUESTIONS