

General Certificate of Education  
June 2004  
Advanced Subsidiary Examination



**MATHEMATICS AND STATISTICS  
(SPECIFICATION B)  
Unit Pure 3**

**MBP3**

Wednesday 23 June 2004 Afternoon Session

**In addition to this paper you will require:**

- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 45 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

**Information**

- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

**Advice**

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

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1 (a) Find the inverse of the matrix  $A = \begin{bmatrix} 3 & -5 \\ 4 & 1 \end{bmatrix}$ . (2 marks)

(b) Use the inverse matrix to solve the simultaneous equations

$$\begin{aligned} 3x - 5y &= 11 \\ 4x + y &= 7 \end{aligned} \quad (3 \text{ marks})$$

2 (a) A curve has equation  $y = \frac{3x + 4}{1 - 2x}$ .

(i) Find the coordinates of the points where the curve crosses the coordinate axes. (2 marks)

(ii) State the equations of its asymptotes. (2 marks)

(iii) Sketch the curve. (2 marks)

(b) Calculate the  $x$ -coordinate of the point where the curve  $y = \frac{3x + 4}{1 - 2x}$  intersects the line  $y = 1$ . (2 marks)

(c) Hence, or otherwise, solve the inequality  $\frac{3x + 4}{1 - 2x} \leq 1$ . (3 marks)

3 The roots of the quadratic equation  $x^2 + (7 + p)x + p = 0$  are  $\alpha$  and  $\beta$ .

(a) Write down the value of  $\alpha + \beta$  and the value of  $\alpha\beta$ , in terms of  $p$ . (2 marks)

(b) Find the value of  $\alpha^2 + \beta^2$  in terms of  $p$ . (2 marks)

(c) (i) Show that  $(\alpha - \beta)^2 = p^2 + 10p + 49$ . (2 marks)

(ii) Given that  $\alpha$  and  $\beta$  differ by 5, find the possible values of  $p$ . (3 marks)

- 4 (a) Find the modulus and argument of the complex number  $-1 + \sqrt{3}i$ , giving the argument in terms of  $\pi$ . *(4 marks)*
- (b) Prove that  $(-1 + \sqrt{3}i)^3$  is real. *(3 marks)*
- (c) Given that  $-1 + \sqrt{3}i$  is a root of the equation  $z^3 + k = 0$ , where  $k$  is real:
- (i) state the value of  $k$ ; *(1 mark)*
- (ii) write down another complex root of the equation  $z^3 + k = 0$ . *(1 mark)*

5 The matrices  $A$  and  $B$  are such that

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

- (a) Find the matrix product  $AB$ . *(2 marks)*
- (b) The transformation  $T$  is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = M \begin{bmatrix} x \\ y \end{bmatrix}$$

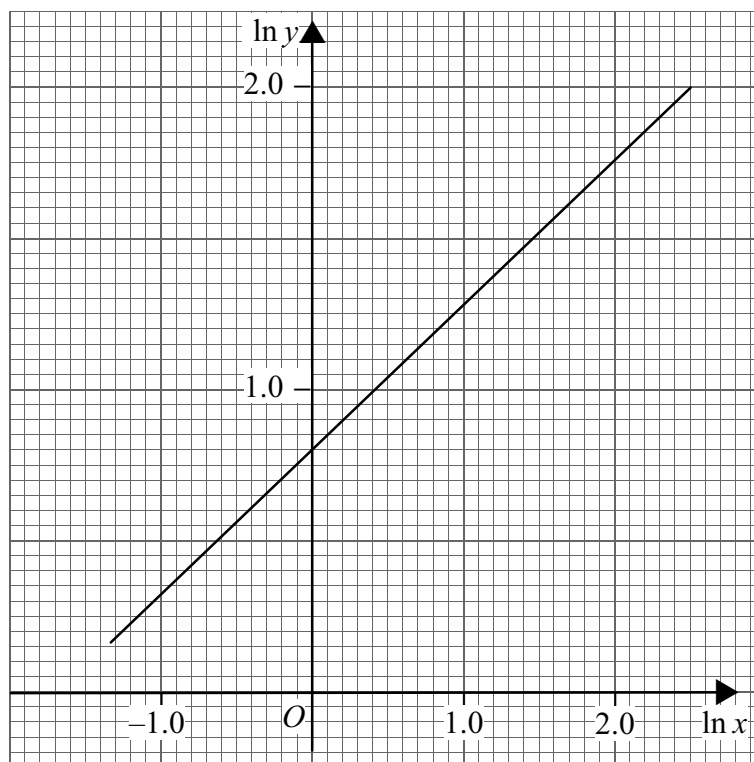
Describe fully the geometrical transformation represented by  $T$  for each of the following cases.

- (i)  $M = A$  *(2 marks)*
- (ii)  $M = B$  *(2 marks)*
- (iii)  $M = AB$  *(2 marks)*

**TURN OVER FOR THE NEXT QUESTION**

6 A student performs an experiment and records data for two variables  $x$  and  $y$ .

Values of  $\ln x$  and  $\ln y$  are calculated and a line of best fit is drawn as shown below.



- (a) Use this graph to find the value of  $y$  when  $x = 3.0$ , giving your answer to two significant figures. *(3 marks)*
- (b) The student believes there is a relationship between  $x$  and  $y$  of the form  $y = Ax^n$ , where  $A$  and  $n$  are constants.
- (i) Express  $\ln y$  in terms of  $\ln A$ ,  $\ln x$  and  $n$ . *(1 mark)*
  - (ii) Use the graph to estimate the values of  $A$  and  $n$ , giving your answers to two significant figures. *(4 marks)*

7 (a) Show that  $\frac{4}{(k+2)(k+3)} - \frac{4}{(k+2)} \equiv \frac{-4}{(k+3)}$ . (2 marks)

(b) Prove by mathematical induction that, for all positive integers  $n \geq 1$ ,

$$\sum_{r=1}^n \frac{4}{(r+1)(r+2)} = 2 - \frac{4}{(n+2)} \quad (4 \text{ marks})$$

(c) Hence find:

(i)  $\sum_{r=3}^n \frac{4}{(r+1)(r+2)}$ ; (2 marks)

(ii)  $\sum_{r=3}^{\infty} \frac{4}{(r+1)(r+2)}$ . (1 mark)

8 (a) (i) Describe a geometrical transformation that maps the curve with equation

$$x^2 + y^2 = 1$$

onto the curve with equation  $(x-1)^2 + y^2 = 1$ . (2 marks)

(ii) Sketch the curve with equation  $(x-1)^2 + y^2 = 1$ . (2 marks)

(b) Show that the curve with cartesian equation

$$(x-1)^2 + y^2 = 1 + 2\sqrt{x^2 + y^2}$$

has the equation  $r = 2 + 2 \cos \theta$ , where  $[r, \theta]$  are polar coordinates. (3 marks)

(c) For the curve with equation  $r = 2 + 2 \cos \theta$ , where  $[r, \theta]$  are polar coordinates:

(i) state the greatest and least values of  $r$ ; (2 marks)

(ii) sketch its graph for  $-\pi < \theta \leq \pi$ . (3 marks)

**TURN OVER FOR THE NEXT QUESTION**

9 The binary operation  $\otimes$  is defined for all integers  $a$  and  $b$  by

$$a \otimes b = a + b - 4$$

- (a) Determine, with a reason, whether the set of integers is closed under  $\otimes$ . *(2 marks)*
- (b) Find the identity element,  $e$ . *(2 marks)*
- (c) Find the inverse of the element  $x$ . *(2 marks)*
- (d) Prove that  $\otimes$  is associative. *(3 marks)*

**END OF QUESTIONS**

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