General Certificate of Education
January 2005
Advanced Subsidiary Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) **MBP3**
Unit Pure 3

Thursday 27 January 2005 Afternoon Session

In addition to this paper you will require:
- a 12-page answer book;
- an insert for use in Question 5 (enclosed);
- a ruler;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 45 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Fill in the boxes at the top of the insert. Make sure that you attach this insert to your answer book.

Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) The complex number z has modulus $3\sqrt{2}$ and argument $\frac{3\pi}{4}$.

Express z in the form $a + b\mathrm{i}$, where a and b are integers. *(2 marks)*

(b) Given that $w = -1 + \mathrm{i}\sqrt{3}$, show that:

(i) $w^2 + 2w$ is real; *(2 marks)*

(ii) $w - \frac{4}{w}$ is purely imaginary. *(3 marks)*

2 Solve the inequality $\frac{3x+1}{3-x} > 2$. *(4 marks)*

3 The matrix $\boldsymbol{M}$ is $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

(a) Find:

(i) $\boldsymbol{M}^2$;

(ii) $\boldsymbol{M}^4$. *(3 marks)*

(b) The transformation $\mathbf{T}$ is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \boldsymbol{M} \begin{bmatrix} x \\ y \end{bmatrix}$$

Describe fully the geometrical transformation represented by $\mathbf{T}$. *(2 marks)*

(c) A reflection in the y-axis is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \boldsymbol{N} \begin{bmatrix} x \\ y \end{bmatrix}$$

Find the matrix $\boldsymbol{N}$. *(2 marks)*

4 The roots of the quadratic equation $2x^2 - 4x + 1 = 0$ are α and β.

(a) Without solving the equation:

(i) write down the value of $\alpha + \beta$ and the value of $\alpha\beta$; *(2 marks)*

(ii) find the value of $\alpha^2 + \beta^2$; *(2 marks)*

(iii) hence find the value of $(\alpha + 3)^2 + (\beta + 3)^2$. *(2 marks)*

(b) Determine a quadratic equation with integer coefficients which has roots $\dfrac{\alpha + 3}{\beta + 3}$ and $\dfrac{\beta + 3}{\alpha + 3}$. *(4 marks)*

5 [An insert is provided for use in this question.]

The variables x and y satisfy a relationship of the form $y = a \times b^x$, where a and b are constants.

Measurements of y for given values of x are shown in the table.

x	0.1	0.2	0.3	0.4	0.5
y	3.09	3.53	4.03	4.61	5.26

(a) Express $\ln y$ in terms of $\ln a$, $\ln b$ and x. *(1 mark)*

(b) (i) Complete the table on the insert, giving values of $\ln y$ to 3 decimal places, and plot $\ln y$ against x on the axes provided. *(3 marks)*

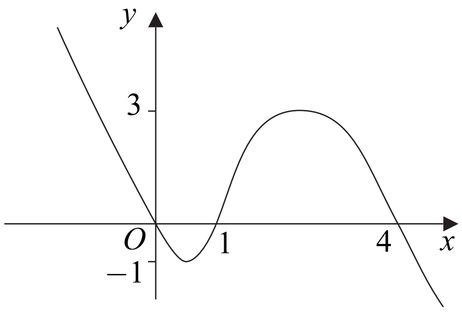
(ii) Draw a suitable straight line to illustrate the relationship between the data. *(1 mark)*

(c) Hence estimate:

(i) the value of y when $x = 0.34$, giving your answer to 3 significant figures; *(2 marks)*

(ii) the values of a and b, giving your answers to 2 significant figures. *(4 marks)*

6 The graph of $y = \mathrm{f}(x)$ is sketched below.



(a) Sketch the graph of $y = |\mathrm{f}(x)|$. *(2 marks)*

(b) Sketch the graph of $y = \dfrac{1}{\mathrm{f}(x)}$ and state the equations of its asymptotes. *(5 marks)*

7 Part of the Cayley table for the set $S = \{1, 3, 5, k, 11, 13\}$ under multiplication modulo 14 is given below. For example, $3 \times 11 = 33 \equiv 5 \pmod{14}$.

	1	3	5	k	11	13
1	1	3	5		11	13
3	3		1		5	a
5	5	1	11		13	b
k						
11	11	5	13			c
13	13					d

The set S is closed under multiplication modulo 14.

(a) Find the values of a, b, c and d in the column headed 13. *(2 marks)*

(b) State the value of k. *(1 mark)*

(c) Given that the identity element is 1, find the inverse of 3. *(1 mark)*

(d) Find a solution of the congruence $13x \equiv 3 \pmod{14}$. *(1 mark)*

(e) Using the value of k found in part (b), show that $k^2 \equiv k + 2 \pmod{14}$. *(2 marks)*

8 (a) (i) Describe the geometrical transformation that maps the circle with equation

$$x^2 + y^2 = 4$$

onto the circle with equation $(x-2)^2 + y^2 = 4$. *(2 marks)*

(ii) Show that the circle with cartesian equation $(x-2)^2 + y^2 = 4$ has equation $r = 4\cos\theta$, where $[r, \theta]$ are polar coordinates. *(4 marks)*

(b) For the curve with equation $r = 8\cos^2\theta$, where $[r, \theta]$ are polar coordinates:

(i) state the greatest and least values of r; *(2 marks)*

(ii) sketch its graph. *(3 marks)*

(c) The curve with equation $r = 8\cos^2\theta$ intersects the circle with equation $r = 4\cos\theta$ at the pole and at the points P and Q. Find the polar coordinates of P and Q, giving the values of θ in terms of π. *(4 marks)*

9 (a) Prove by mathematical induction that, for all positive integers n,

$$\sum_{r=1}^{n} 4(r+1)(r+2)(r+3) = (n+1)(n+2)(n+3)(n+4) - 24$$ *(6 marks)*

(b) (i) Show that

$$\frac{1}{(r+1)(r+2)} - \frac{1}{(r+2)(r+3)} = \frac{2}{(r+1)(r+2)(r+3)}$$ *(1 mark)*

(ii) Use the method of differences to prove that

$$\sum_{r=1}^{n} \frac{2}{(r+1)(r+2)(r+3)} = A - \frac{1}{(n+2)(n+3)}$$

and state the value of the constant A. *(4 marks)*

(iii) Hence find $\displaystyle\sum_{r=1}^{\infty} \frac{2}{(r+1)(r+2)(r+3)}$. *(1 mark)*

END OF QUESTIONS

Surname		Other Names								
Centre Number						Candidate Number				
Candidate Signature										

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Insert for use in Question 5.

Fill in the boxes at the top of this page.

Attach this insert securely to your answer book.

TURN OVER

x	0.1	0.2	0.3	0.4	0.5
y	3.09	3.53	4.03	4.61	5.26
$\ln y$