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General Certificate of Education

Mathematics and Statistics 6320
Specification B

MBP3 Pure 3

Mark Scheme
2005 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
√or ft or F		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
–x ee		deduct x marks for each error
pi		possibly implied
sca		substantially correct approach

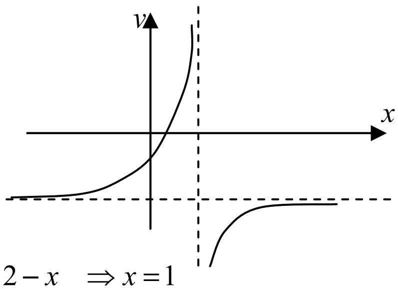
Abbreviations used in Marking

MC – x	deducted x marks for mis-copy
MR – x	deducted x marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

Application of Mark Scheme

No method shown:	
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

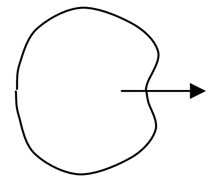
Mathematics and Statistics B Pure 3 MBP3 June 2005

Q	Solution	Marks	Total	Comments
1(a)(i)	$\left(\frac{3}{4},0\right)$ and $\left(0,-\frac{3}{2}\right)$	B1 B1	2	
(ii)	Asymptote at $x=2$ and at $y=-4$	B1 B1	2	
(iii)		M1 A1	2	One branch of hyperbola roughly correct Good graph
(b)	$4x-3=2-x \quad \Rightarrow x=1$ Using graph, $x<1$ Also $x>2$	M1 B1 B1	3	M0 for $(4x-3)<(2-x)$ or $(4x-3)(2-x)<(2-x)^2$ M1 $5(2-x)(x-1)<0$ or quotient A1 Hence $x<1, x>2$ A1
	Total		**9**	
2 (a)(i)	$\alpha+\beta=3$; $\alpha\beta=5$	B1 B1	2	
(ii)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=9-10=-1$ Since $\alpha^2+\beta^2<0$, α,β not both real	M1 A1√ E1√	3	
(iii)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ $= 27-45 = -18$	M1 A1	2	Or $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$ etc **ag**
(b)	$\sum \text{roots} = \frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}$ $=-\frac{18}{5}$ $x^2-\left(-\frac{18}{5}\right)x+5(=0)$ $5x^2+18x+25=0$	M1 A1 M1 A1	4	Formation of quadratic using sum of roots and attempt at product of roots $=\alpha\beta=5$ cso (integer coefficients and = 0)
	Total		**11**	

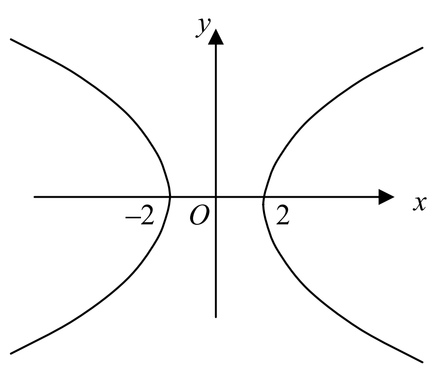
MBP3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	det **M** = $5k - 14$	B1	1	
(b)	$\lvert\det \mathbf{M}\rvert = 1$	M1		Condone det **M** = 1
	$\Rightarrow k = 3$	A1		
	or $k = 2.6$	A1	3	
(c)(i)	$\boldsymbol{M}^{-1} = \frac{1}{6}\begin{bmatrix} 5 & 7 \\ 2 & 4 \end{bmatrix}$	M1 A1	 2	Condone a "pair" of slips in matrix, or multiplication by/ omission of det **M** for M1
(ii)	$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 5 & 7 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \end{bmatrix}$	M1		Must premultiply by $\mathbf{M}^{-1}$ ft their inverse
	$x = 9$,	A1		
	$y = 5$	A1	3	Coords are (9,5)
	Total		**9**	
4(a)	$\frac{4 \pm \sqrt{16-52}}{2}$	M1		Use of quadratic equation formula or completing square
	$\sqrt{-36} = 6\text{i}$ or $\sqrt{-9} = 3\text{i}$	B1		
	$\Rightarrow x = 2 \pm 3\text{i}$	A1	3	
(b)	$(p + 3\text{i})^2 = p^2 + 6p\text{i} - 9$	B1		
	Comparing real/imag parts	M1		$6p = 12$ or $p^2 - 9 = q$
	$p = 2$; $q = -5$	A1	3	
	Total		**6**	
5	When $n = 1$; LHS $= \frac{1}{2}$;RHS $= 2 - \frac{3}{2} = \frac{1}{2}$;	B1		(True when $n = 1$)
	Assume formula true for $n = k$ Add $(k+1)$th term to both sides	E1		Plus the conclusion; hence true …
	namely $\frac{k+1}{2^{k+1}}$	B1		
	RHS $= 2 - \frac{k+2}{2^k} + \frac{k+1}{2^{k+1}} = 2 - \frac{***}{2^{k+1}}$	M1		$2 -$ attempt at common denominator
	$= 2 - \frac{\{2k + 4 - k - 1\}}{2^{k+1}} = 2 - \frac{k+3}{2^{k+1}}$	A1	5	
	Result true for $n = k+1$ Hence true for n=1, 2, 3 etc by induction			Must have conclusion to earn E1 mark above
	Total		**5**	

MBP3 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	Maximum value of $r = 5$	B1		
	when $\theta = \pi$	B1		condone angles mod 2π
	Minimum value of $r = 1$	B1		
	when $\theta = 0$	B1	4	
(ii)	Symmetry about $\theta = 0$	B1		
	Correct graph – approx 5: 1 ratio	B1	2	
(b)(i)	$8c^2 + 2c - 3 = 0$			
	$\Rightarrow (2c-1)(4c+3) = 0$	M1		Attempt to factorise or solve quad eqn
	$\cos\theta = \frac{1}{2}, \quad \cos\theta = -\frac{3}{4}$	A1	2	
(ii)	Use of $r = 3 - 2\cos\theta$ to find r	M1		or using $r = 8\cos^2\theta$
	$\left[2, \frac{\pi}{3}\right], \left[2, -\frac{\pi}{3}\right], \left[\frac{9}{2}, \cos^{-1}(-0.75)\right]$	A1√		one pair of matching r and θ ft
		A1√		second pair of matching r and θ ft
	$\left[\frac{9}{2}, -\cos^{-1}(-0.75)\right]$	A1	4	All 4 points correct
	Total		**12**	
7(a)(i)	£15 000	B1	1	
(ii)	£5 000	B1	1	
(b)(i)	$\frac{dV}{dt} = -2\,500t^{-\frac{1}{2}}$	M1		
		A1		
	When $t = 4$; $\frac{dV}{dt} = -1\,250$	A1	3	cso
(ii)	Car is depreciating (at this instant in time)	E1√		ft increasing in value if > 0 in (b) (i)
	at a rate of £1 250 per year	E1√	2	
(c)(i)	$\log V = \log a + \log b^{-t}$	M1		One rule of logs used properly
	$= \log a - t \log b$	A1	2	
(ii)	$\log 11\,500 = \log a - \log b$ and			
	$\log \; 5\,000 = \log a - 4 \log b$	B1√		or $11\,500 = a / b$ & $5\,000 = a / b^4$
	$3 \log b = \log(115/50)$ or $3 \log a = \log$ **	M1		or $b^3 = 2.3$
	$b = 1.32$	A1		
	$a = 15\,200$	A1	4	Condone 15 180
	Total		**13**	

MBP3 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$p \otimes e = p \quad \Rightarrow p + e + pe = p \Rightarrow e = 0$	B1	1	or $p \otimes 0 = p + 0 + 0 = p$ etc
(b)	$p \otimes 3 = p + 3 + 3p$	M1		or $3 \otimes q = 3 + q + 3q$
	$p \otimes 3 = 0$	M1		or $3 \otimes q = 0$
	$4p = -3$; Hence $p = -\frac{3}{4}$	A1	3	
(c)(i)	$(p \otimes q) \otimes r$			
	$= (p + q + pq) + r + (p + q + pq)r$	M1		full marks at this stage if correct
	$= p + q + r + pq + qr + rp + pqr$	A1	2	
(ii)	$p \otimes (q \otimes r)$ considered	M1		
	$= p + (q + r + qr) + p(q + r + qr)$			
	Shown to equal $(p \otimes q) \otimes r$			
	yes, it is associative	A1	2	
	Total		**8**	
9(a)		M1		Hyperbola one branch correct or two half branches correct
		B1		(2,0) and (–2,0) marked or stated
		A1	3	good symmetrical hyperbola
(b)	Translation	M1		
	through $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$	A1		Move left by one etc scores M1, A0
	Stretch in y- direction	M1		
	Scale factor 2	A1	4	
	Total		**7**	
	TOTAL		**80**	